EVALUATING MATHEMATICS KNOWLEDGE FOR TEACHING AS AN OUTCOME OF PRE-SERVICE TEACHER EDUCATION: LESSONS FROM THE HIGH PERFORMING COUNTRIES IN THE TEDS-M STUDY

Maria Teresa Tatto
Michigan State University
Michael Rodriguez
University of Minnesota

Abstract

We investigate how diverse policies have shaped pre-service teacher education programs across high achieving countries and search for patterns that may begin to explain results obtained in our assessments of mathematics teaching knowledge among future primary and secondary teachers. Using data from the recently released IEA TEDS-M study, descriptive analysis and multilevel modeling, we present evidence that in high achieving countries pre-service teacher education has been able to produce highly knowledgeable teachers, a result correlated with quality assurance strategies and opportunities to learn mathematics knowledge for teaching provided to future teachers. In some cases however, notably in the USA, other factors such as future teachers' gender and socioeconomic background seem to have a stronger influence than teacher education arrangements.

For more than a decade a number of national and international reports have recommended that resources be directed toward a better understanding of what teachers need to know and do to teach mathematics successfully (e.g., National Commission . . . , 2000; National Research Council, 2001; Kilpatrick, Swafford, & Findell, 2001). While research has begun to advance our understanding of the mathematical knowledge considered most important for primary school mathematics teaching, we know much less about the knowledge most important for teaching secondary school mathematics or how these findings are reflected in the teacher education curricula (see, e.g., Baumert et al., 2010; Hill, Sleep, Lewis, & Ball, 2007; Schmidt, Blömeke and Tatto, 2011). A recent review commissioned by the National Academy of Sciences in the United States concluded that: “Successful mathematics teachers need preparation that covers knowledge of mathematics, of how students learn mathematics, and of mathematical pedagogy that is aligned with the recommendations of professional societies” and recommends that “both quantitative and qualitative data about the programs of study in mathematics offered and required at teacher preparation institutions is needed, as is research to improve understanding of what sorts of preparation approaches are most effective at developing effective teachers” (Committee on the Study of Teacher Preparation Programs in the United States, 2010, p. 123 and p. 124).

The TEDS-M study begins to answer these questions. The Teacher Education Study in Mathematics (TEDS-M) is the first collaborative effort of worldwide institutions to study the mathematics preparation of future primary and secondary teachers. The study explores if what future teachers learn in pre-service teacher education leads to more effective knowledge of mathematics and mathematics for teaching as measured by knowledge assessments. The TEDS-M study relies on rigorous methodologies, nationally representative samples and large scale surveys of teacher education institutions, faculty, and future teachers to provide valid and reliable information on the organization and outcomes of preparing future teachers to teach mathematics at the primary and secondary levels. The study collected data in 2008-2009, and was directed by Michigan State University (MSU) in collaboration with the Australian Council for Educational Research (ACER), , the IEA, and National Research Centers in 17 countries including Botswana, Canada, Chile, Chinese Taipei, Georgia, Germany, Malaysia, Norway, Philippines, Oman, Poland, Russia, Singapore, Spain, Switzerland, Thailand and the USA. The study was sponsored by the IEA and funded by the National Science Foundation.

In this article we present an investigation of the links between future primary and secondary teachers' mathematics teaching knowledge and a number of features of pre-service teacher education programs using data from TEDS-M in those countries that show high levels of performance in the IEA's TIMSS mathematics tests, and in the PISA tests. Using multilevel modeling we show that the effects of teacher education vary across countries and, within countries, across programs and that this variation is related to program selectivity and opportunities to learn offered to future teachers. We shows the links between performance in the TEDS-M assessments of mathematics teaching knowledge and varied opportunities to learn in courses and argue that the particular characteristics of these opportunities to learn as they are implemented in different programs and contexts need to be explored more deeply as they represent a "missing construct in the development of high quality teachers". The article includes countries in which pre-service teacher education has been able to produce highly knowledgeable teachers capitalizing in selection strategies and on the opportunities to learn that seem to exert a powerful influence on future teachers' mathematics knowledge for teaching. In some cases such as the case of public teacher education institutions in the USA however individual characteristics of future teachers such as previous preparation, gender, and socioeconomic background seem to have a stronger effect than teacher education program arrangements. We discuss the implications of these findings in the context of recent calls for rigorous curriculum standards and suggest future research directions based on what we have learned from TEDS-M.

Evaluating the Outcomes of Teacher Education: The TEDS-M Study

The idea of quality[1] permeates the policy discourse in education. The implicit definition of quality among the teacher education programs participating in the TEDS-M study was dependent

[1] The standard of something as measured against other things of a similar kind; the degree of excellence of something; a distinctive attribute or characteristic possessed by someone or something http://oxforddictionaries.com/definition/quality [consulted 11/12/2011].

on the levels of performance as demonstrated by future teachers in our knowledge assessments. Because TEDS-M is a groundbreaking study its comparative methodology facilitated finding standards of quality as national systems of teacher education are compared against each other. Importantly TEDS-M is a collaborative self-study designed and implemented by mathematics teacher educators seeking to improve their own practice.

TEDS-M's primary purpose was to gather empirical evidence about mathematics teacher preparation for primary and lower secondary grades. The data were collected in sixteen countries during 2008-2009 from a national representative sample of institutions, teacher educators and future teachers who were in their last year of their teacher preparation and were considered ready to begin to teach (see Author et al. in press, or Author 2011 for more detail) [2].

Countries and Demographics

In this article we report on findings from future primary and secondary school teachers and their programs in "high achieving countries" in mathematics as indicated by international assessments such as Chinese Taipei, Germany, Poland, Russian Federation, Singapore, Switzerland and contrast these results with the USA. The data comes from the TEDS-M future teacher survey and from the survey of teacher education programs. The future teacher survey consisted of questions asking about background characteristics and opportunities to learn, and an assessment of mathematics knowledge for teaching which measured mathematics content knowledge and mathematics pedagogy content knowledge. The survey of teacher education programs consisted of questions asking information about the organization and content of the programs included in the study. Most of the countries included in this article have a relatively

[2] In the collaborative tradition of IEA, the countries invite themselves to participate in IEA studies. For TEDS-M a total of 15,163 future primary teachers were surveyed in 451 institutions and 9,389 future secondary teachers were surveyed in 339 institutions in 16 countries participated in the TEDS-M study (see Author et al., in press, for the TEDS-M final report).

high or very high Human Development Index (HDI)[3], ranging from .755 at the lower end to .905 at the higher end, and all have a well-established system of pre-service teacher education (see Table 1). The lowest GDP per capita are in the Russian Federation and Poland, yet the latter invests a high percentage of its GDP per capita in education, second only to the USA and Switzerland among this group of countries.

The USA, Germany, and Switzerland have on average higher levels of schooling among adults, while Singapore has the lowest, followed by the Russian Federation and Poland. The income index (GNI) per capita is lower in Poland and the Russian Federation. The lowest human development index (HDI) was found in the Russian Federation and the USA, and the highest in Germany.

Methods[4]

Populations, samples and program characteristics

In TEDS-M, the populations of interest included institutions where future primary and secondary teachers receive their preparation to teach mathematics, the teacher educators who prepared them in mathematics/mathematics pedagogy and general pedagogy (not included in this article), and the future teachers in their last year of training. The international sampling plan used a stratified multi-stage probability sampling design. The targeted individuals (in this case future teachers) were randomly selected from a list of in-scope future teachers for each of the randomly selected teacher preparation (TP) institutions. In smaller countries, all teacher preparation institutions were selected to participate in TEDS-M, and in some countries, all eligible future

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[3] The Human Development Index (HDI) is a measure of life expectancy, education, and income used to compare countries worldwide (see United Nations Development Programme 1999).
[4] A detailed description of the TEDS-M methods is in Author (in press).

teachers in the sampled institutions were surveyed. While the samples are of unequal sizes these should be seen as representative of national systems of teacher education in the countries.[5]

Program characteristics as shown in Table 2 must be seen as expressions of national policy, which are a response to global calls for improving the quality of teaching, for instance, column five shows that in all countries there are governance and accountability systems of varying strengths. All these programs have been created with the explicit goal to provide future teachers with the knowledge they are expected to have to become effective teachers of mathematics. All countries offer pre-service education to future primary and secondary teachers in universities in response to the more complex levels of knowledge required of teachers in recent times.

The duration of the programs across countries reveals some variation, but it is no shorter than 3 years, and most fluctuate between 4 and 5 years. In all cases these countries have programs that are exclusively dedicated to train teachers for either primary or secondary levels and with the exception of Chinese Taipei and the Russian Federation there is a high degree of differentiation within the primary and secondary programs.

Table 2 shows that teacher preparation programs vary by the intended grade levels and the extent to which they prepare generalists (teachers who are expected to teach all subjects of the primary or lower secondary school curriculum) and mathematics specialists. All countries in the table have separate programs to prepare primary and secondary teachers and even within these two levels, some have more specialized programs to educate teachers according to lower

[5] The minimum sample size was set at 50 institutions per level; and an effective sample size of 400 future teachers per level in a given country. "Effective sample size" means that the sample design must be as efficient (i.e., precise) as a simple random sample of 400 teachers from a (hypothetical) list of all eligible future teachers. Deviations from the desired 100% participation rates appear in Tables A1-A4 in the Appendix.

and upper grades within the primary and secondary levels[6]. The table shows that there is not such thing as a program to train primary teachers but rather four "primary program types": lower primary, primary, primary/secondary and primary specialists; similarly there are two "secondary programs types": lower secondary and lower and upper secondary. One hypothesis of this study is that program structure determines opportunities to learn, consequently understanding the differences in types of programs will be important in understanding the results of our analysis.

Instruments

To assess knowledge among future teachers, the TEDS-M research team used items developed by the Pre-TEDS-M study known as MT21, and solicited items from the Knowing Mathematics for Teaching Algebra (KAT) Project at Michigan State University, the Learning Mathematics for Teaching (LMT) Project at the University of Michigan, from researchers in Australia, and from the participating countries. The TEDS-M, MSU team developed items as well. At each stage of the item development process, expert panels examined the content validity and appropriateness of the items. These reviews took into consideration clarity, correctness, cultural relevance, classification within the framework of domains and sub-domains, relevance to teacher preparation, and curricular level. In preparation for the field trial, detailed manuals were written for all aspects of the study (sampling, survey administration and procedures, etc.). Scoring guides and rubrics were prepared for all constructed response items, and sample responses were collected to provide a basis for training. Two scoring training sessions were carried out in preparation for the field trial and two more in preparation for the main study.

For the questionnaires, every question used in TEDS-M was reviewed by area and psychometric experts, and field tested repeatedly in the participating countries before the main

[6] The variability in offerings in teacher education in the USA makes it impossible to describe all programs by name or by curriculum offerings, instead the program "names" in Table 2 are types of programs such as primary, lower secondary and secondary.

study. Indices were analyzed for psychometric quality, including the provision of internal-consistency evidence, score reliability evidence, and particularly evidence of measurement invariance. Only those items and questions that performed well according to confirmatory factor analysis were included in the final instruments. The instruments were translated from the English to the local languages and back translated to confirm accuracy and consistency. Further details on the methods and design of the main study can be found in the TEDS-M Conceptual Framework and in the Technical Report (Author 2008, and Author in press). The content and reliability of the scores and scales[7] are described below.

Future Teachers Measures

Knowledge for Teaching: Mathematics Content Knowledge and Mathematics Pedagogy Knowledge Assessments

Teachers' professional knowledge has been conceptualized in a variety of ways throughout the years, yet an important departure came in the 1980s with the re-conceptualization of the complex kinds of knowledge that teachers need to be able to teach well. In an influential article, Shulman (1987) argued that there is a "knowledge base for teaching" that could be understood as a combination of content knowledge, pedagogical content knowledge, general pedagogical knowledge, curriculum knowledge, knowledge of learners, knowledge of educational contexts, and knowledge of educational ends, purposes and values (p.8). Four of these concepts find global resonance in the international literature and provide a common analytical framework: (a) *content knowledge* (CK), seen as the set of accumulated "knowledge, skills and dispositions that are to be learned by school children" (p. 8-9) but which also has a base in the disciplines and in ideas about what it means to know in those content areas; (b)

[7] The reliabilities for the OTL and beliefs scales are unweighted and were estimated using jMetrik 2.1 (Meyer, 2011). The reliability estimates are based on the congeneric measurement model, which allows each item to load on the common factor at different levels and allows item error variances to vary freely (each item can be measured with a different level of precision). This is the most flexible measurement model and most appropriate for measures with few items.

general pedagogical knowledge, which includes "broad principles and strategies of classroom management and organization" (p.8); (c) *pedagogical content knowledge* (PCK), which includes a "blending of content and pedagogy into an understanding of how particular topics, problems, or issues are organized, represented and adapted to the diverse interests and abilities of learners and presented for instruction" (p.8), and (d) *curricular knowledge,* which involves an understanding of how the particular subject to be learned is organized sequentially into a coherent program of study.

While agreement as to the knowledge base for teaching can be seen as an important achievement, developing measures to assess it has proven a difficult undertaking (see Hill, Sleep, Lewis, and Ball 2007) and requires attention to the context as well as institutional and cultural aspects of teaching and learning to teach (see Boero, Dapueto, and Parenti 1996).

In TEDS-M, the test of Mathematics Content Knowledge [MCK] consisted of four domains: number and operations, algebra and functions, geometry and measurement, and data and chance. The assessment framework for mathematics content follows closely that used in the Trends in Mathematics and Science Studies [TIMSS] (see Mullis, Martin, and Foy 2007; Garden, Lie, Robitaille, Angell, Martin, Mullis, Foy and Arora 2006). The test for Mathematical Pedagogical Content Knowledge [MPCK] was primarily informed by the framework used by the Mathematics Teaching in the 21st Century Project (MT21), a study of mathematics teacher preparation for lower secondary grades in six countries that was originally designed to be a precursor to TEDS-M (Author 2011). Three domains were measured in this test: curricular knowledge, knowledge of planning for teaching, and knowledge of enacting teaching. Five blocks of items were assembled for the primary test and three for the secondary test, each with 12 – 15 questions. Each future teacher received a booklet with two of the blocks of items about knowledge for teaching mathematics. The test was designed to take up to 60 minutes of time under a controlled administration. To sample all the domains we wanted to measure, we developed the assessment using a "matrix sampling" design (Mazzeo, Lazer, and Zieky 2006).[8]

To obtain comparable estimates of performance, item response theory (IRT) was used. Item response theory allows estimates of performance to be obtained on the same scale even when the set of items taken by each individual is different (see, e.g., De Ayala 2009). The first step in the process for forming the reporting score scales was to calibrate the test items and then evaluate the results to determine if the data were well fit by the IRT models. Items with poor fit and items that showed other violations of assumptions of the models were carefully reviewed. Some of these items were removed from the computation of the reported scores. Others required modifications to the scoring procedures such as combining score categories on items with multiple score points. After this review and revision process, the sets of items were calibrated again using weights so that each country contributed equally to the calibration (Wu, Adams, Wilson, and Haldane 2007). The final calibration results were used to estimate the location of the examinees on a common IRT scale and were then transformed so that the international mean for the calibration sample on each of the MCK and MPCK scales was 500 and the international standard deviation was 100. In addition TEDS-M developed anchor points and anchor point descriptions to give tangible meanings to points reached by respondents on the reporting score scales (see Author 2011).[9] These anchor points provide the standard against which levels of performance among the participating future teachers is measured.

[8] More details on the test and the study in general can be found in Author et al. 2008; in Author et al. in press, and in Author in press.

[9] Appendix 1 shows these results for both tests by country and by specific program type for primary and secondary levels. The tables A1-A4 include estimates of the proportion of future teachers in the sample from each country who reached or exceeded each anchor point (AP). Consequently, for the entire cohort of future teachers in the sampled target population and for the particular program types, we have empirical evidence of performance levels (e.g., for MCK with probabilities greater than .70 (AP1) or less than .50 (AP2) for MCK; for MPCK with probabilities

Reliability: For the international sample, the reliability for the mathematics content knowledge and the mathematics pedagogical content knowledge scores for the future primary teachers were.83 and .66 respectively; and the reliability for the mathematics content knowledge and the mathematics pedagogical knowledge scores for the future secondary teachers were .91 and .72 respectively.[10]

Background and prior attainment

In addition to the knowledge assessment, each form of the Future Teacher Questionnaire consisted of questions about the background of respondents (socioeconomic status or SES[11], age, gender, and prior attainment). We thought it likely that we will see higher levels of knowledge among future teachers in countries which also show pupils with high levels of achievement in international tests; in other words teacher education may not be able to overcome the deficiencies of previous schooling.

Program Measures

TEDS-M conducted a survey of teacher preparation (TP) institutions to collect data on institutional program characteristics. Program information was also collected from the future teachers. "Program" referred to the set of courses or units of study and other learning activities that constituted the formal preparation provided to future primary or lower-secondary teachers. These were the programs from which future teachers were sampled to complete the TEDS-M Future Teacher Questionnaire. Program measures include questions about the program

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between .50 and .70 (AP) for MPCK) based on what they were projected, from the IRT model, to be able to do or not do within the specified probabilities.

[10] Reliabilities tend to be high if there is a lot of variation in the sample relative to the size of the standard error. The reliability will be low if one of the following occurs: (a) There is a small standard deviation in the sample or (b) there is a large standard error (e.g., the test was too easy for a particular sample), (see Author in press).

[11] Using principal components analysis, a scale was created to obtain a proxy measure of SES, by averaging the possessions in the parents or guardians home variables (such as number of books at home, father's highest level of education and mother's highest level of education).

opportunities to learn, program coherence and philosophy or orientation. We describe these below.

Opportunities to Learn

Opportunities to learn or OTL can be simply defined as time allowed for learning (see Floden 2002). For TEDS-M we assumed that what future teachers come to know may be in great part determined by the opportunities to learn as represented in the curriculum and other learning experiences provided to them as part of their pre-service teacher education. In addition, and especially in the case of future teachers, opportunities to learn also occur outside teacher education programs and can be mediated by the program's structure and philosophy. For instance it is possible that programs' lack of emphasis on academic and school mathematics may be compensated by mathematics requirements as a condition for admittance to teacher education (see Author, for an example).

Previous research has used the concept of opportunity to learn (OTL) as central to explaining the impact of teacher preparation on teacher learning (Author 1993; Author 1996). In addition previous research has shown that it is not enough to provide opportunities to learn but that these need to be internally coherent if they are to have an effect (see Author 1996; 1998; 1999). These studies taken together have shown that coherent program offerings seem to contribute to graduates the knowledge, skills, and dispositions that are consistent with expected program goals.

The TEDS-M study includes a number of indices to allow exploration of the opportunities to learn that future mathematics teachers have across countries. These include counts of topics studied on opportunities to learn university or tertiary level mathematics, and on school level mathematics. All the other OTL indices are based on a 4-point scale (e.g., expressing frequency such as "never" to "often") and include questions on opportunities to lean: mathematics education/pedagogy; education /pedagogy; accommodations to classroom diversity and reflections on practice; from school experience and the practicum; in a coherent teacher education program. Based on a series of confirmatory factor analyses, OTL scales were scaled using the Rasch model and the results were rescaled to be centered at the point on the scale that is associated with the middle of the rating scale (essentially neutral). All OTL scales are based on a score scale where 10 is located at the neutral position.

Four opportunity to learn indices aggregated at the program level were significantly related to our measures of knowledge and are included in our models: (a) geometry taught at the tertiary level, which included topics such as foundations of geometry or axiomatic geometry, analytic/coordinate geometry, non-Euclidean geometry, and differential geometry[12]; (b) upper school level mathematics, including functions, relations, equations, data representation, probability, statistics, calculus, and validation, structuring, and abstracting; (c) readings in mathematics education and pedagogy, including readings about research on mathematics, on mathematics education, and on teaching and learning, and analysis of examples of teaching; and (d) program coherence, which included questions such as whether the program seemed to be planned to meet the main needs of future teachers in each stage of preparation, whether later courses in the program built on what was taught in earlier courses, whether the program was organized in a way that covered what effective teachers needed to learn, whether the courses seemed to follow a logical sequence of development in terms of content and topics, whether each of the courses was clearly designed to prepare future teachers to meet a common set of explicit

[12] While geometry may seem an isolated topic among many that can be studied in university level mathematics, geometry builds on a number of mathematics abilities that are considered important to develop in-depth mathematics understandings.

standard expectations for beginning teachers, and whether there were clear links between most of the courses in the teacher education program.

Reliability: Regarding the OTL topics indicators, since they are counts of courses taken, there are no reliability estimates because they are not measures, only counts. The Fit Indices provide evidence that the groupings of the courses, based on logical organization of courses as judged by experts, make sense given the data as reported by future teachers.[13] These are for tertiary level mathematics for the primary sample CFI .911, TLI .954, RMSEA .044; and for the secondary sample CFI .969, TLI .986, RMSEA .032. For school level mathematics for the primary sample CFI .97, TLI .973, RMSEA .057; and for the secondary sample CFI .892, TLI .846, RMSEA .085.

The reliabilities for the opportunity to do class reading on research on mathematics teaching and learning, is for the primary sample .85, and .83 for the secondary sample. For the opportunity to learn in a coherent program, the reliability is .96 for the primary sample, and .97 for the secondary sample.

Program Philosophy

When programs' philosophy is clearly articulated and coherently shared by program faculty (fundamentally becoming an institutional norm), it seems to have an effect on whether or not future teachers change from naïve to professional views as defined by their programs (see Author 1996, 1998, 1999b). Asking future teachers to report on their beliefs about teaching and learning provides indicators that help us understand how, as a group, future teachers think about

[13] Comparative Fit Index (CFI): The CFI depends in large part on the average size of the correlations in the data. If the average correlation between variables is not high, then the CFI will not be very high. An acceptable model is indicated by a CFI larger than .93 (Byrne, 1994), but .85 is acceptable (Bollen 1989).The Tucker Lewis index (TLI) is relatively independent of sample size (Marsh, Balla, and McDonald 1988). Values over .90 or.95 are considered acceptable (e.g., Hu and Bentler 1999). Root Mean Square Error of Approximation (RMSEA): Another test of model fit, good models are considered to have a RMSEA of .05 or less. Models whose RMSEA is .1 or more have a poor fit.

what they know, and what they believe (see research by Sternberg 1992, cited in Shepard, 2001; Author 2003). Their views together can be seen as a reflection of the program's philosophy.

Measurements of beliefs have been developed to evaluate whether teachers have a student-centered or a direct transmission view of teaching (see Fennema and Franke, 1992; and Staub and Stern 2002), and whether mathematics can be seen as a formal, structural, procedural, or applied subject (Grigutsch, et al. 1998; Ingvarson et al. 2005, 2007); these views are considered essential in regulating the teaching and learning of mathematics.

TEDS-M measured beliefs using 6-point rating scales (e.g., "strongly agree to strongly disagree") in five different areas; two of these scales are relevant to our current exploration: (a) beliefs about the nature of mathematics; and (b) beliefs about learning mathematics. The items in beliefs about the nature of mathematics included questions that explore how future teachers perceive mathematics as a subject (e.g., mathematics as formal, structural, procedural, or applied). The items in beliefs about learning mathematics included questions about the appropriateness of particular instructional activities, questions about students' cognition processes, and questions about the purposes of mathematics as a school subject.

Based on a series of confirmatory factor analyses, belief scales were scaled using the Rasch model and the results were rescaled to be centered at the point on the scale that is associated with the middle of the rating scale (essentially neutral). All belief scales are based on a score scale where 10 is located at the neutral position.

Reliability: For the international sample the reliability for the beliefs scale "mathematics as a set of rules and procedures" for future primary teachers is .94, and for future secondary teachers .93; and for "learning mathematics through active involvement" for the primary future teachers is .92, and for the secondary future teachers .92.

Methods of Analysis

We first present the descriptive results of the study followed by the results of multivariate modeling to explore the complex interaction of programs' and future teachers' characteristics on teacher education outcomes. Because the countries participating in TEDS-M vary by the size of their population we used both regression and HLM to model these interactions. We describe these methods of analysis before proceeding to present the results of the study.

Regression Analysis

Multiple regression is a general linear model that allows for multiple variables (predictors) on which a continuous outcome variable is regressed. The multiple variables explain variation in the outcome variable, each controlling for the effect of the others. Our interest is to obtain the conditional relation between explanatory variables and outcomes (MCK, PCK), which can be standardized to facilitate comparison across variables - allowing us to identify those explanatory variables that are most highly associated with the outcome variables. The overall model fit index of interest is the proportion of variance accounted for (R-squared) by the combined effects of the explanatory variables. We used regressions to analyze the data from the "smaller" countries participating in TEDS-M.

Hierarchical Linear Modeling Analysis

HLM is a statistical method that helps us compute regressions at multiple levels. The analysis in this study uses a two-level HLM model in which future teachers were nested within their teacher education programs[14] within their countries. Heuristically, HLM estimates a regression within each program and combines them to see if there is a common regression across

[14] This is similar to students nested within classrooms however the reader must have in mind that teacher education is fundamentally higher education, students or future teachers have multiple classrooms and have multiple teacher educators. Thus the nesting unit in TEDS-M is the program.

programs within a given country. When regression slopes show variation across programs, researchers can examine program-level characteristics that may explain such variation. This method is useful for evaluating program factors and their effects on future teachers' outcomes. In addition HLM partitions variance components across levels and provides an estimate of variance in future teacher performance that exists within programs and between programs in a given country. For instance, in this case the variance in one of our outcome measures (e.g., mathematics content knowledge or MCK) is partitioned into level-1 variance (within programs) and level-2 variance (between programs). In HLM we can run an unconditional model, which is a model without explanatory variables that helps us to answer the question, "how much variance in future teachers' outcome can be attributed to factors on which programs differ systematically from one another?" Using the level 1 and level 2 variances respectively from the unconditional model, we can compute the % of variance explained from the predictors in the final model. Typical results of multilevel models yield 10%-33% of the variance between programs (the intra-class correlation coefficient, ICC), according to Taylor, Pearson and Rodriguez (2003). In classroom studies some have found as much as 25%-50% of the variance between classrooms (Frank, 1998). The results in the analysis of the full model, which include explanatory variables, will tell us the percent of variance explained within programs (or level-1) and also the percent of variance explained between programs (or level-2).

The coefficients in the model (which have been standardized within each country) are expected to explain variation or effect in the outcome. If the resulting effect, called a standardized beta or β, is 0.33, for example (see Table 5, Poland MCK column), we would interpret this as meaning that a change of one standard deviation in the predictor variable (opportunity to learn functions, probability and calculus) is associated with a 0.33 standard deviation change in the outcome variable (mathematics content knowledge).

Because of the improved estimation methods employed by HLM, including the use of maximum likelihood and Bayes estimates, interpretation of statistical results can be broadened to include a larger *p*-value in the statistical tests such as *p*-values at or near .10 (see Bryk and Raudenbush 1992, 32-56). When the model is theoretically based and relations are consistent with prior research, using a significance criterion of .10 allows us to continue examining important variables, because our primary goal is to learn from the data. There is precedence for this practice in multilevel modeling in educational research (Rodriguez, Taylor, Pearson, and Peterson 2005). In the following sections we present the results of the study. We used *HLM* to analyze the data from the "larger" countries participating in TEDS-M.

Who Are the Future Teachers of Mathematics and What Do They Know?

Background and Past Performance

Among future primary teachers (in Tables 4 and 5), those with higher socioeconomic status are in Germany and Switzerland, and in the Russian Federation and in the USA. Future primary teachers are in their early to late twenties, with the younger group in Chinese Taipei, Switzerland, Poland, and the USA and the oldest in Singapore and Germany. Most primary future teachers are female, with the highest proportions in Germany and Poland (.95) and the lowest (.72) in Chinese Taipei. Self-reported levels of attainment as per average grades in high school (with 1 "below average for year level," and 5 "always at top of year level") placed future primary teachers as above average with those in the USA (3.49), the Russian Federation (3.37), Chinese Taipei (3.22), and Singapore (3.03), reporting higher grades. Below average or just slightly above average were those reporting lower grades in Germany (2.53), Poland (2.80), and

Switzerland (2.88). It is clear that on average teacher education at the primary level does not attract those who are "always at the top" of their year level.

Future secondary teachers (in Tables 4 and 5) are similar in socioeconomic background and average age at time of graduation as their future primary counterparts. While still dominated by females, in Poland (.81), the Russian Federation (.72) and the United States (.69), the pattern shifts in Chinese Taipei, Singapore and Switzerland all showing a larger proportion of males than is the case among the primary group (with proportions of .38, .47, and .41 respectively) . The self-reported level of attainment as per average grades in secondary school was of 3.28 or above, with the highest average attainment reported by future secondary teachers in the USA and the Russian Federation (3.88 and 3.80 respectively out of 5), and the lowest in Poland (3.28) and Germany (3.32). While this group of future secondary teachers does not on average report being "always at the top" of their year level they do report a higher level of attainment than that reported by their primary level counterparts.

Mathematics Content Knowledge (MCK) and Mathematics Pedagogy Knowledge (MPCK)

The scores obtained by future primary and secondary teachers on the assessments of mathematics (MCK) and mathematics pedagogy knowledge (MPCK) per country, are in Tables 4 and 5 respectively. Table 4 shows the results for the high scoring, smaller countries in the TEDS-M sample, and Table 5 for the larger countries.

Table 4 shows that the future teachers at the primary level in Chinese Taipei show the highest scores (623 in MCK and 592 in MPCK), followed by Singapore (590 in MCK and 593 in MPCK), Switzerland (543 in MCK and 537 in MPCK), and Germany (510 in MCK and 499 in MPCK). Among the secondary programs in this group, future teachers from Chinese Taipei again show the highest scores (666 in MCK and 647 in MPCK), followed by Singapore (573 in

MCK and 554 in MPCK), Germany (541 in MCK and 553 in MPCK), and Switzerland (530 in MCK and 546 in MPCK), most above the international mean (500, SD 100).

Table 5 shows that among the three largest high scoring countries in TEDS-M with programs preparing future primary teachers, the highest average level of knowledge in mathematics and mathematics pedagogy was in the Russian Federation (536 in MCK and 511in MPCK), followed by the USA (515 in MCK and 543in MPCK), and Poland (476 in MCK and 467 in MPCK). Among programs preparing future secondary teachers, the highest average level of knowledge in mathematics and mathematics pedagogy were in the Russian Federation (593 in the mathematics test, and 569 in the mathematics pedagogy test), followed by the USA (536, and 529), and Poland (536, and 526), most above the international mean (500, SD 100).

In sum, while most of the primary and secondary future teachers score above the international mean (500) the performance of future teachers in Chinese Taipei and Singapore is significantly different from the rest of the countries.

What Characterizes Teacher Education Programs?

Opportunities to Learn

TEDS-M developed a number of different indicators of opportunities to learn. The first we present here have to do with topics studied (with values ranging from 0-4); the rest have to do with the character or organization of these opportunities to learn (presented as scales which are centered at 10 representing neutral). Regarding topics studied, future primary teachers had low to above average exposure to mathematics courses, including university level mathematics, and mathematics of the school curriculum with the lowest proportion of topics in Germany and Singapore among the small countries (see Table 4); and the USA followed by Russia and Poland among the large countries (see Table 5). Future primary teachers in Switzerland and Chinese Taipei receive the most exposure. The frequency with which future primary teachers engaged in reading research on teaching and mathematics was particularly low in all the small countries with the exception of Singapore; and the Russian Federation and the USA among the larger countries. Primary future teachers in Germany gave the lowest coherence mark to their programs followed by Switzerland, all the other programs received higher coherence ratings.

Future secondary teachers in contrast with their primary level counterparts have a much higher exposure to university and school level mathematics topics, especially in Chinese Taipei among the small samples (see Table 4); and Poland, and Russia among the larger samples close to or above 3.5 (see Table 5). Singapore where future secondary teachers enter the program with substantial mathematics knowledge, shows the lowest exposure to tertiary level mathematics but a higher exposure to school level mathematics, and this is the pattern across all the programs in both the small and large samples with the exception of Russia where emphasis in both tertiary and school mathematics is somewhat balanced and relatively high (3.8 and 3.4 respectively out of 4).

The frequency with which future secondary teachers engaged in reading research on teaching and mathematics was particularly low across all the countries with small samples; and in Poland among the larger countries. Overall this practice does not seem affected by whether programs are preparing primary or secondary teachers. Program coherence was seen as higher in Singapore, Chinese Taipei, the Russian Federation, and the USA.

Program Philosophy

What teachers think about mathematics as a subject and how it is best learned is an important area of concern across teacher education. Two scales representing these views at the program level had a clear link with the tests results, the view that mathematics is a collection of rules and procedures, and the view that mathematics is better learned through active learning (both scales are centered at 10 representing neutral). On average and consistent with widely accepted views on learner-centered teaching (e.g., teachers must focus on what the learner is thinking when learning—and not solely on the subject/lesson to be taught), most primary and secondary future teachers tend to agree with the idea that mathematics requires active learning. There is less agreement with the first view indicating that mathematics can be seen as a collection of rules and procedures, a view that if upheld would imply a more traditional view of mathematics and if rejected a more learner-centered view signaling a philosophy more attuned to current world thinking in education. Across the board, Germany, Poland, the Russian Federation, Switzerland and the USA future teachers are in disagreement with the more traditional view while Chinese Taipei and Singapore future teachers tend toward agreement while also agreeing with the view that mathematics is better learned through active learning (see Tables 4 and 5). The Asian countries' view that these views may not be opposite to each other but rather complementary point to a likely false dichotomy often encountered in Western ideas about learning.

Program Wealth

Programs' wealth is a scale created by measures of future teachers' home possessions including number of books at home, and parents' levels of education. The higher levels among primary future teachers are in Germany and Switzerland and in the Russian Federation followed by the USA. The same can be said of the secondary programs.

What is the Relationship between Future Teachers' Characteristics, Program Characteristics and Teacher Education Outcomes?

To answer this question for the smaller countries participating in TEDS-M we used regression analysis. Table 6 shows the results of our regression analysis for the primary future teachers and Table 7 shows the results of our regression analysis for the secondary future teachers in the four small high achieving countries. Variables that have a significant correlation with the level of performance in our MCK and MPCK assessments are future teachers' background including SES, age, gender, and prior attainment, opportunities to learn mathematics (at the tertiary level and school level) and beliefs. In some cases the level of program wealth was important.

Background Effects on MCK and MPCK Performance

Socioeconomic background had an overall positive but non-significant correlation with future primary teachers' performance in our tests. Regarding age, younger future primary teachers show higher levels of achievement in our tests. One standard deviation decrease in age in Chinese Taipei, Singapore and Switzerland is associated with 0.08, 0.10, and 0.07 respectively standard deviation increase in their score in MCK. This association is stronger in PCK in Switzerland and Singapore (one standard deviation decrease in age is associated with 0.12 and 0.17 standard deviation increase in MPCK scores). Prior attainment is, as expected, an important predictor of success in our tests, across the board, one standard deviation increase in prior attainment in Chinese Taipei, Singapore and Switzerland is associated with 0.18, 0.19, 0.14, and 0.23 standard deviation increase respectively in MCK, and 0.16, 0.20, 0.13, and 0.14 in MPCK. Socioeconomic background had a positive correlation with future secondary teachers' performance in our tests: one standard deviation increase in SES in Germany and Singapore is associated with 0.10, and 0.22 respectively standard deviation increase in their score in MCK, and with 0.11 and 0.12 in Singapore an Switzerland. Regarding age, with the exception of Germany, younger future secondary teachers show higher levels of achievement in our tests. One standard deviation decrease in age in Singapore and Switzerland is associated with 0.26 and 0.16 increase in their score in MCK, and with a 0.18 increase in MPCK in Singapore. In Germany one SD change in age is associated with 0.13 increase in their scores in the MCK test. As was the case with the primary future teachers, females seem to do worse than males in Chinese Taipei, Germany and Singapore in the MCK tests, and in the MPCK test in Singapore and Switzerland. Prior attainment is, as expected, was an important predictor of success in our MCK tests, across the board, one standard deviation increase in prior attainment in Chinese Taipei, Singapore and Switzerland is associated with 0.11, 0.24, 0.14, and 0.24 standard deviation increase respectively in MCK, and 0.17, 0.09, and 0.13 in MPCK.

Program Effects on MCK and MPCK Performance

In the primary programs in Germany and Switzerland opportunities to learn university level mathematics proved to be an important predictor of success in our MCK test where one standard deviation increase in the number of topics studied in this domain was associated with 0.27, and 0.12 standard deviation increase respectively in MCK. Regarding school mathematics the relationship was positive but not significant in Germany; however in the other countries the relationship is negative likely because they do not study many of these topics in their teacher preparation program and the same was the case concerning reading mathematics research in their courses.

In the secondary programs in Chinese Taipei, opportunities to learn university level mathematics proved to be an important predictor of success in our MCK test where one standard deviation increase in the number of topics studied in this domain was associated with 0.27 SD increase in the score of MCK and 0.30 standard deviation increase respectively in MPCK. Regarding school mathematics one standard deviation increase in the number of topics studied in this domain in Chinese Taipei was associated with 0.09 increase in the score of MCK, and in Germany one standard deviation increase in the number of topics studied in this domain was associated with 0.37 SD increase in the score of MCK, and 0.27 increase in the score of MPCK.

Other program factors such as programs' philosophy or orientation had important roles. For the primary programs, the level of program coherence was an important predictor of success on MCK in Switzerland. The view of whether mathematics teaching and learning could be considered as learning a series of rules and procedures or as conceptual active learning is an important predictor of the scores obtained on our tests in some countries. For instance one standard deviation increase in the agreement that mathematics can be taught as procedural had a 0.13 decrease in the scores obtained in the MCK test in Germany, and 0.11 of a SD decrease in the MPCK test in Switzerland.

Among those future teachers in the secondary programs, the view of whether mathematics teaching and learning could be considered as learning a series of rules and procedures was negatively associated with the scores obtained on our MCK and MPCK tests in Germany (0.15). In Chinese Taipei, and different from their primary level counterparts, one standard deviation increase in the agreement that mathematics can be taught as active learning was associated with a 0.08 increase in the scores obtained in the MPCK test.

Overall, program wealth had a positive effect on the performance of future primary teachers across the countries studied; these effects were significant in Germany (one standard deviation increase in program wealth was associated with .08 to 0.10 increase in the knowledge scores for MCK and MPCK respectively.

Among future secondary teachers, program wealth had a positive effect on their performance across the countries studied; these effects were significant and positive regarding our assessment of MPCK in Chinese Taipei.

For the larger countries, we used a two-level hierarchical linear modeling or HLM (Raudenbush and Bryk 2002; Raudenbush et al. 2004) to investigate the influence of teacher education on future primary and secondary teachers' mathematics knowledge for teaching (MCK and MPCK). We used the same HLM model across all countries but analyzed each country and level (primary and secondary) separately. To avoid repetition we present these results in a single Table 8. The models we used are detailed in Appendix 2.

The variance between programs in each outcome is expressed by intra-class correlations or ICCs. Higher ICCs mean that within a country there are important differences in teacher education programs and that these differences in performance are mostly between programs rather than within programs; in other words some programs tend to produce highly knowledgeable future teachers in mathematics, while others do not. In addition these differences can be attributable to programs rather than to the individual characteristics of future teachers. Because of the lower reliabilities in the mathematics pedagogy knowledge test, the HLM estimates are lower.

Table 8 (left half) shows the HLM model for future primary and secondary teachers and programs. The ICCs at the bottom of Table 8 indicate that programs have different influences on the knowledge that future teachers attain at the end of their pre-service teacher education across countries. For the primary future teachers in the mathematics content test (MCK), the higher ICCs are in Poland, Russia, and the USA (with 60%, 40%, and 16% of the variance attributed to programs). For the mathematics pedagogy test (MPCK), the higher ICCs are in Poland, and Russia, (with 39%, and 37% of the variance attributable to differences between programs), as in all these cases the proportion of variance explained is higher between programs than within programs. The results for the USA for both outcomes suggests that very little variation in performance on MCK and MPCK is due to programs, as programs performed similarly (ICCs=5% and 5% respectively).

Table 8 (right half) shows the HLM model for future secondary teachers and programs. As with the primary programs the ICCs show a wide range of variance between programs. As with the primary data, because of the lower reliabilities in the mathematics pedagogy knowledge test, there are lower HLM estimates in the MPCK area. For the mathematics content test (MCK), the higher ICCs are in Russia, followed by the USA, and Poland, (with 50%, 38%, and 30% respectively). For the mathematics pedagogy test (MPCK), the higher ICCs are in Russia, Poland, and the USA (with 26%, 24%, and 17% respectively). In all these cases the proportion of variance explained is higher between programs than within programs.

Background Effects on MCK and MPCK Performance

Among the primary future teachers in Table 8 there is a small but significant relationship between higher socioeconomic levels (SES) and higher levels of performance in the mathematics content knowledge (MCK) test in Poland, and in both tests (MCK and MPCK) in the USA. Regarding age and gender, older and female future teachers had slightly lower scores across the countries. Prior attainment in schooling had a larger positive association with the scores obtained in both tests as expected across all countries.

Among the secondary future teachers in Table 8 we found that higher levels of achievement in our tests are related to higher socioeconomic levels (SES) but this relationship is not significant. As in the case with primary teachers, younger teachers across all countries performed better in our tests, and this relationship is significant in Poland, and the USA. Higher levels of achievement in our tests were more common among future male teachers; this relationship was significant for the MCK test in Poland, and the USA; and for the MPCK test for Poland and the USA. Only in the Russian Federation did females have higher scores than males in our tests, and this result was statistically significant. As expected, prior attainment levels were positively and strongly related to higher test results in both tests in all countries, with the exception of Poland in the MPCK test.

Program Effects on MCK and MPCK Performance

Opportunities to learn university level mathematics in primary programs had significant associations with performance in our tests. For instance in Poland, future teachers in programs providing opportunities to learn university level mathematics (one SD higher) scored about one-tenth of a SD higher on MCK and MPCK. Future primary teachers in programs that covered a larger number of topics on school mathematics specifically topics in the areas of functions, probability and calculus (one SD higher) also scored higher in our tests in Poland (1/3 of a SD higher in MCK, and 1/5 of a SD higher in MPCK), and in Russia (1/4 of a SD higher in MCK). Having the opportunity to read research connected with mathematics teaching and learning had a mixed effect on future teachers' performance on our tests. There was a strong and positive link between the level of program coherence (one SD higher) and high scores on both tests (about 1/3 of a SD higher in MCK and 1/4 of a SD higher in MPCK) among future teachers in the Russian Federation.

For the secondary programs, the number of topics studied in university level geometry had a positive effect on the MCK test results. Future secondary teachers in programs offering a higher number of topics in Geometry at the university level (one SD higher) score about 1/10 to 1/15 of a SD higher in the MCK test in Poland, and the Russian Federation, while in the USA the difference was even higher, about 1/3 of a SD. Opportunities to learn university level geometry (one SD higher) also affected performance in our MPCK test with future teachers in the Russian Federation, and the USA scoring about 1/15 of a SD higher. The number of topics covered in school level mathematics (one SD higher) —specifically function, probability and calculus—had an overall positive effect on future teachers' performance in our tests, with future secondary teachers in the USA scoring above 1/10th of a SD higher in the MPCK test. Future secondary teachers in programs that emphasized reading research connected with mathematics teaching and learning (one SD higher) scored higher in our tests in Russia (1/5 of a SD higher in MCK and ¼ of a SD higher in MPCK). The level of program coherence had a negative effect on scores in the USA (specifically in the MPCK test).

The philosophy or orientation of the programs preparing primary future teachers characterized by a view of mathematics as a collection of rules and procedures had a negative relationship with high levels of achievement in both tests. Future primary teachers in programs with a stronger belief (one SD higher) that mathematics is a collection of rules and procedures score lower in our tests in Poland (about one-tenth of SD lower in MCK and MPCK), in Russia (about one-third of SD lower in MCK, and about one-fifteenth of SD lower in MPCK), and in the USA (about one-tenth of SD lower in both MCK and MPCK). Conversely, there was a general positive relationship between performance in our tests and the view that mathematics is better learned through active learning; specifically, future primary teachers in programs that held that view in the USA scored about 1/10 of SD higher in the MCK test.

For the secondary future teachers, as was the case among primary programs, there was a general negative relationship between high levels of achievement in both tests and the view of mathematics as a collection of rules and procedures. Future secondary teachers in programs with a stronger belief (one SD higher) that mathematics is a collection of rules and procedures scored lower in our tests in Poland (about 1/5 of SD lower in MPCK), in Russia (about one-third of SD lower in MCK, and about one-tenth of SD lower in MPCK), and in the USA (about 1/5 of SD lower in MCK and ¼ of a SD lower in MPCK). The view that mathematics is better learned through active learning received a weak endorsement among future teachers in these secondary level programs.

Overall, program wealth had a positive effect on the performance of future primary teachers across the countries studied; these effects were significant in Poland, and the USA.

Regarding the future secondary teachers, program wealth had a positive but not significant effect on their performance across the countries studied.

Discussion

Our study shows that the design, curricular content and orientation of teacher education programs can have substantial effects on the level of knowledge that future teachers are able to acquire. In general, programs where future teachers are more successful in our assessments have rigorous standards in selecting those who enter the program, they have a demanding and sequential (versus repetitive) curriculum, frequent formative evaluations (written and oral), and stringent graduation requirements. A conceptual, problem solving and active learning orientation seems to characterize the views of mathematics among those future teachers who score higher in our assessments, likely reflecting the way they themselves learned mathematics and the views that the programs espouse (see author 2012).

Consequently, teacher education programs can increase their effectiveness by selecting future teachers according to their characteristics (e.g., previous school performance) and strengthening formative and summative evaluation as they progress through their program. In fact previous performance in school, gender and socioeconomic status are characteristics that seemed to explain in some degree the knowledge that future teachers demonstrate at the end of their formal initial formal initial teacher education

A general conclusion of our analysis is that future teachers, who did well in their previous schooling, and specifically in high school, perform better in our mathematics knowledge for teaching assessments. In all countries, opportunities to learn university level mathematics, mathematics of the school curriculum, and reading research on teaching and mathematics were positively related to future teachers' knowledge as measured in our assessments. The more traditional view of mathematics as a finished product has given way to a more contemporary view of mathematics as a process of inquiry and to the idea that mathematics is better learned through active learning (Ernest, 1989, p. 250). This seems to be especially true among future teachers who will teach the lower grades. In general, successful programs seemed to be more coherently organized around the idea of what effective teachers need to know.

For primary programs, the most important positive influence of teacher education in mathematics knowledge for teaching is the opportunity to learn school level mathematics, specifically in the areas of function, probability and calculus. Another important yet negative influence on knowledge as measured by our assessment, was found among future teachers who as a group hold the view that can be summarized as "mathematics is a collection of rules and procedures that prescribe how to solve a problem" and that "mathematics involves the remembering and application of definitions, formulas, mathematical facts and procedures," this is a view that stands in contrast with the more accepted view, supported by cognitive science research on learning that, "in addition to getting a right answer in mathematics, it is important to understand why the answer is correct" and that "teachers should allow pupils to figure out their own ways to solve a mathematical problem." While the first is a view that may be espoused by teacher education programs, it could also be a "naïve view" held by future teachers based on commonly held "cultural norms" and which remains unchallenged and unchanged by their program. In other words the program may end up reinforcing traditional ways of teaching and learning, already acquired by future teachers in their own schooling (author, 1999).

For secondary programs the most important influence on knowledge for teaching is the opportunity to learn university level mathematics, specifically geometry, and the opportunity to read research on teaching and learning. As in the primary programs the view that "mathematics is a collection of rules and procedures that prescribe how to solve a problem" had a negative influence on performance in our assessment.

One conclusion of this study is that teacher education programs' quality of opportunities to learn—as measured by their association with high levels of mathematics teaching knowledge, coherence on program philosophy and approaches, and moderate quality assurance and accountability mechanisms, are all features that seem to contribute to increased levels of mathematics knowledge for teaching among future teachers. While the TEDS-M study is limited in how much it can tell us about the effects of high quality teacher education on initial teaching practice, it provides the basis for the development of further inquiry into this unexplored yet essential question: what elements contribute to the development of high quality teachers?

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Table 1 Basic Demographics and Education Data for the Countries Reported[15]

	Population Total (millions)[16]	GDP per Capita (2008 ppp US$)[17]	Expenditure on education (% of GDP)[18]	Mean Years of Schooling (adults)[19]	Income Index (GNI per capita)[20]	HDI[21]
Chinese Taipei	22.9[22]	32,800	4.2	--	--	0.895
Germany	82.3	32,255	4.5	12.2	.838	0.905
Poland	38.1	16,705	4.9	10.0	.739	0.795
Russian Fed.	141.4	14,561	3.9	9.8	.713	0.755
Singapore	4.6	45,978	3.0	8.8	.897	0.866
Switzerland	7.5	36,954	5.2	11.0	.858	0.840
USA	311.7	41,761	5.5	12.4	.869	0.771

A dash (–) indicates comparable data are not available.

[15] Data comes from United Nations Development Programme Human Development Report (2010), Country Profiles [http://hdr.undp.org/en/data/profiles/], unless otherwise indicated.

[16] The de facto population in a country, area or region as of 1 July of the year indicated in this case 2010.

[17] Sum of value added by all resident producers in the economy plus any product taxes (less subsidies) not included in the valuation of output, calculated without making deductions for depreciation of fabricated capital assets or for depletion and degradation of natural resources. When expressed in purchasing power parity (PPP) US$ terms, it is converted to international dollars using PPP rates. An international dollar has the same purchasing power over GDP that the U.S. dollar has in the United States. Chinese Taipei Data (2009) comes from CIA World Factbook. https://www.cia.gov/library/publications/the-world-factbook/geos/tw.html

[18] Total public expenditure (current and capital) on education expressed as a percentage of GDP.

[19] Average number of years of education received by people ages 25 and older in their lifetime based on education attainment levels of the population converted into years of schooling based on theoretical durations of each level of education attended.

[20] The Gini coefficient is a measure of the deviation of the distribution of income (or consumption) among individuals or households within a country from a perfectly equal distribution. A value of 0 represents absolute equality, a value of 100 absolute inequality.

[21] The Human Development Index or HDI Is a composite measure of three basic dimensions of human development: health, education and income 2011.

[22] Taiwan data is based on CIA World Factbook. https://www.cia.gov/library/publications/the-world-factbook/geos/tw.html

Table 2 Primary and Secondary Teacher Preparation Programs by Country and Grade Span

Country	Name given to program	Duration (years/ +2nd phase)	Grades prepared to teach /as generalist/specialist	Strength of governance and accountability systems***
Chinese Taipei	Elementary Teacher Education	4.5	1-6 / generalist	Strong
	Secondary Mathematics Teacher Education	4.5	7-12 /specialist	
Germany	Teachers for Grades 1-4 with Mathematics as Teaching Subject (Type 1a)	3.5+2.0	1-4 / generalist	Moderate/Strong
	Teachers for Grades 1-4 without Mathematics as Teaching Subject (Type 1b)	3.5+2.0	1-4 / generalist	
	Teachers of Grades 1-9/10 with Mathematics as Teaching Subject (Type 2a)	3.5+2.0	1-9/10 / specialist in 2 subjects	
	Teachers for Grades 1-10 without Mathematics as Teaching Subject (Type 2b)	3.5+2.0	1-4 / generalist	
	Teachers for Grades 5/7-9/10 with Mathematics as Teaching Subject (Type 3)	3.5 +2.0	5/7-9/10 / specialist in 2 subjects	
	Teachers for Grades 5/7-12/13 with Mathematics as a Teaching Subject (Type 4)	4.5+2.0	5/7-12/13 / specialist in 2 subjects	
Poland*	B. Pedagogy, integrated teaching (first cycle)	3	1 to 3 / generalist	Moderate/Strong
	M.A. Integrated Teaching, Long Cycle	5	1 to 3/generalist	
	Mathematics BA (First cycle)	3	4-9/specialist	
	Mathematics (Long cycle)	5	4-12/specialist	
Russian Federation	Primary Teacher Education	5	1 to 4 / generalist	Moderate/Strong
	Teacher of Mathematics	5	5-11 / specialist	

Table 2 Primary and Secondary Teacher Preparation Programs by Country and Grade Span (cont.)

Country	Name given to program	Duration (years/ +2nd phase)	Grades prepared to teach /as generalist/specialist	Strength of governance and accountability systems***
Singapore	PGDE, Primary Option C	4+1	1-6 / generalist	Strong
	BA (Ed) (Pri)	4	1-6 / generalist	
	BSc (Ed) (Pri)	4	1-6 / generalist	
	Dip Ed, Primary Option A	2	1-6 / specialist in 2 subjects	
	Dip Ed, Primary Option C	2	1-6 / generalist	
	PGDE, Primary Option A	4+1	1-6 / specialist	
	PGDE, Lower Secondary	4+1	7-8 / specialist in 2 subjects	
	PGDE, Secondary	4+1	7-12 / specialist in 2 subjects	
Switzerland	Teachers for Grades 1-2/3	3	1-2/3 / generalist	Moderate
	Teachers for Primary School (Grades 1-6)	3	1-6 / generalist	
	Teachers for Primary School (Grades 3-6)	3	3-6 / generalist	
	Teachers for Secondary School (Grades 7-9)	4.5	7-9 / generalist, some specialization	
USA	Primary Teacher Education	4	1 to 3-4-5 / generalist	Moderate / low
	Primary Teacher Education**	4+1	1 to 3-4-5/ generalist	
	Primary and Secondary Teacher Ed.	4	4/5-8/9 / specialist	
	Primary and Secondary Teacher Ed. **	4+1	4/5-8/9 / specialist	
	Secondary	4	6/7-12/ specialist	
	Secondary **	4+1	6/7-12/ specialist	

*Some opt to enroll in the program part time and some full time.
**Denotes consecutive programs (e.g., bachelors degree required before entering the pre-service teacher education program). All other programs are concurrent.
*** Defined by the strength of "quality assurance" procedures to regulate entry into teacher education (control over supply of teacher education students, selection standards for entry into teacher education), accreditation of teacher education programs, and entry to the teaching profession (see Author et al, in press, chapter 2).

Table 3 TIMSS 2007 and PISA 2009 Mathematics Results, and TEDS-M Results for Selected Countries Participating in TEDS-M[1]

	TIMSS 2007 Grade 4	TIMSS 2007 Grade 8	PISA 2009	TEDS-M MCK Scaled mean Score[1]		TEDS-M MPCK Scaled mean Score[1]		TEDS-M MCK Scaled mean Score[1]		TEDS-M MPCK Scaled mean Score[1]	
Country	Mathematics Average Score	Mathematics Average Score	Mathematics Average Score	Primary 1	Primary 2	Primary 1	Primary 2	Secondary 1	Secondary 2	Secondary 1	Secondary 2
Chinese Taipei	576*	598*	543*		623		592		667		649
Germany	525	--	513*	501	555	491	552	483	585	515	586
Poland	--	--	495	456	614	452	575	529	549	520	528
Russian Federation	544*	512	468	536		512			594		566
Singapore	599*	593*	562*	586	600	588	604	544	587	539	562
Switzerland	--	--	534*	512	548	519	539	531		549	
United States	529	508	487	518	520	544	545	468	553	471	542

[1]TIMSS scale average score=500 [http://nces.ed.gov/pubs2009/2009001.pdf]; OECD Average Score on the Mathematics Scale=496 [http://www.oecd.org/dataoecd/54/12/46643496.pdf]; TEDS-M Average Score=500

*Average score is higher than US average score (p<.05)

— No data available; Program type not existent in country

Table 4 Means and Standard Deviations for TEDS-M Future Teachers Knowledge for Teaching Mathematics in Primary and Secondary Programs[23]

	PRIMARY PROGRAMS								SECONDARY PROGRAMS							
	CHINESE TAIPEI N=921		GERMANY N=875		SINGAPORE N=376		SWITZERLAND N=924		CHINESE TAIPEI N=355		GERMANY N=620		SINGAPORE N=371		SWITZERLAND N=137	
Future Teachers Primary (Level 1)	**M**	**SD**	**M**	**SD**	**M**	**SD**	**M**	**SD**	**M**	**SD**	**M**	**SD**	**M**	**SD**	**M**	**SD**
MCK score	**623.77**	**84.09**	**510.57**	**86.01**	**590.69**	**73.97**	**543.30**	**65.52**	**666.58**	**75.37**	**541.90**	**84.33**	**573.92**	**60.72**	**530.64**	**48.79**
MPCK Score	**592.48**	**67.91**	**499.74**	**89.43**	**593.22**	**71.20**	**537.61**	**63.46**	**647.66**	**94.46**	**553.11**	**98.16**	**554.84**	**84.68**	**546.47**	**73.03**
SES*[SES]	**-.362**	**.93**	**.527**	**.90**	**-.429**	**.87**	**.231**	**.89**	**-.503**	**.87**	**.415**	**.93**	**-549**	**.79**	**.125**	**.90**
Age [MFA001]	**23.16**	**2.11**	**27.32**	**4.03**	**26.68**	**4.72**	**23.64**	**3.59**	**24.06**	**2.27**	**28.98**	**4.91**	**26.73**	**4.00**	**26.20**	**4.30**
Proportion female [MFA002_]1=F; 0=M	**.72**	**.44**	**.92**	**.26**	**.73**	**.43**	**.85**	**.35**	**.38**	**.48**	**.62**	**.48**	**.47**	**.50**	**.41**	**.49**
Prior attainment: Average grades in secondary school (1=below average for year level; 5=Always at top of year level)	**3.22**	**1.10**	**2.53**	**.85**	**3.03**	**.89**	**2.88**	**.96**	**3.65**	**1.07**	**3.32**	**.88**	**3.51**	**.95**	**3.40**	**.91**
Teacher Education Programs Primary (Level 2)	**M**	**SD**	**M**	**SD**	**M**	**SD**	**M**	**SD**	**M**	**SD**	**M**	**SD**	**M**	**SD**	**M**	**SD**
Average number of university level mathematics topics in geometry ever studied (range 0-4) [MFB1GEOM]	**2.04**	**.14**	**1.05**	**.63**	**1.31**	**.36**	**2.18**	**.18**	**3.22**	**.32**	**2.19**	**.47**	**1.49**	**.40**	**2.78**	**.42**
Average number of school level mathematics topics in function, probability and calculus studied as part of the TE program (range 0-4)	**1.95**	**.19**	**1.07**	**.63**	**1.50**	**.33**	**1.09**	**.39**	**3.45**	**.35**	**2.48**	**.57**	**2.63**	**.30**	**2.89**	**.39**
Average frequency with which future teachers engaged in reading research on teaching and mathematics (scales centered at 10 representing neutral) [MFB5READ]	**8.97**	**.64**	**7.60**	**1.09**	**9.44**	**.44**	**8.89**	**1.05**	**9.69**	**.85**	**8.00**	**.49**	**9.12**	**.15**	**8.75**	**.80**
Average level of program coherence (scales centered at 10 representing neutral)[MFB15COH]	**11.46**	**.37**	**9.27**	**.48**	**12.67**	**.52**	**10.18**	**.42**	**11.96**	**.56**	**9.17**	**.48**	**12.03**	**.17**	**10.44**	**.87**
Average agreement with the belief that mathematics is a collection of rules and procedures (scales centered at 10 representing neutral) [MFD1RULE]	**10.72**	**.14**	**9.98**	**.24**	**11.04**	**.16**	**9.98**	**.20**	**10.80**	**.19**	**9.65**	**.15**	**10.91**	**.06**	**9.84**	**.28**
Average agreement with the belief that mathematics is better learned through active learning (scales centered at 10 representing neutral) [MFD2ACTV]	**12.11**	**.10**	**12.31**	**.41**	**11.74**	**.19**	**12.38**	**.33**	**12.35**	**.25**	**12.43**	**.32**	**11.53**	**.14**	**12.46**	**.43**
Average SES for each program (aggregated from future teachers SES) [SES]	**-.363**	**.10**	**.528**	**.16**	**-.426**	**.05**	**.234**	**.21**	**-.498**	**.19**	**.410**	**.30**	**-.550**	**.10**	**.119**	**.20**

[23] Germany, Singapore and Switzerland have programs that prepare future primary teachers to teach at different levels (e.g., generalists and specialists). In some cases there are differences in their performance which could be attributed to differences in their programs. There were differences in Germany (in MCK generalists scored 501 and specialists 555; in MPCK 491 and 552); in Singapore the differences were smaller (in MCK generalists scored 586 and specialists 600; in MPCK 588 and 604); as were in Switzerland (in MCK lower primary generalists scored 512 and upper primary generalists 548; in MPCK 519 and 539). While we present the average in this table, our regression analysis also indicates which differences among programs contribute to higher scores in our assessments.

Table 5 Means and Standard Deviations for TEDS-M Future Teachers Knowledge for Teaching Mathematics in Primary and Secondary Programs

VARIABLES AND LEVELS	PRIMARY PROGRAMS						SECONDARY PROGRAMS					
	Poland[24] $N_{Level\ 1}$ =1822 $N_{Level\ 2}$ =125		Russian Fed. $N_{Level\ 1}$ =2162 $N_{Level\ 2}$ =49		United States[25] $N_{Level\ 1}$ =1073 $N_{Level\ 2}$ =78		Poland $N_{Level\ 1}$ =247 $N_{Level\ 2}$ =34		Russian Fed. $N_{Level\ 1}$ =1951 $N_{Level\ 2}$ =48		United States $N_{Level\ 1}$ =461 $N_{Level\ 2}$ =68	
Future Teachers (Level 1)	**M**	**SD**	**M**	**SD**	**M**	**SD**	**M**	**SD**	**M**	**SD**	**M**	**SD**
MCK score	476.14	88.91	536.37	94.07	515.73	69.54	536.05	88.91	593.26	90.84	536.13	65.36
MPCK Score	467.74	95.04	511.87	89.06	543.43	67.50	525.98	95.04	569.01	94.67	529.15	80.55
SES*[SES]	-0.29	0.73	0.62	0.70	0.44	0.84	-0.11	0.73	0.60	0.64	0.46	0.84
Age [MFA001]	24.95	5.33	28.88	4.75	25.46	6.45	23.13	5.33	22.01	1.59	25.26	6.45
Proportion female [MFA002_]1=F; 0=M	0.95	0.22	0.93	0.26	0.92	0.28	0.81	0.22	0.72	0.45	0.69	0.28
Prior attainment: Average grades in secondary school (1=below average for year level; 5=Always at top of year level) [MFA009_]	2.80	0.84	3.37	0.92	3.49	1.00	3.28	0.84	3.80	0.89	3.88	1.00
Teacher Education Programs (Level 2)	**M**	**SD**	**M**	**SD**	**M**	**SD**	**M**	**SD**	**M**	**SD**	**M**	**SD**
Average number of university level mathematics topics in geometry ever studied (range 0-4) [MFB1GEOM]	2.29	0.75	2.06	0.45	1.88	0.71	3.23	0.47	3.81	0.21	2.59	0.74
Average number of school level mathematics topics in function, probability and calculus studied as part of the TE program (range 0-4) [MFB2SLMF]	1.80	1.40	2.22	0.50	2.13	0.47	3.82	0.25	3.46	0.32	2.81	0.79
Average frequency with which future teachers engaged in reading research on teaching and mathematics (scales centered at 10 representing neutral) [MFB5READ]	8.12	0.85	10.70	0.88	10.40	1.13	8.15	1.34	10.28	0.75	10.61	1.34
Average level of program coherence (scales centered at 10 representing neutral)[MFB15COH]	11.29	0.97	13.39	0.80	12.97	1.49	11.53	1.14	12.93	0.75	12.78	1.63
Average agreement with the belief that mathematics is a collection of rules and procedures (scales centered at 10 representing neutral) [MFD1RULE]	10.86	0.63	10.72	0.35	11.03	0.47	10.39	0.51	10.52	0.28	10.71	0.59
Average agreement with the belief that mathematics is better learned through active learning (scales centered at 10 representing neutral) [MFD2ACTV]	12.10	0.60	11.77	0.36	12.02	0.66	12.29	0.79	11.89	0.47	12.11	0.90
Average SES for each program (aggregated from future teachers SES) [SES]	-0.25	0.34	0.61	0.21	0.44	0.33	-0.10	0.28	0.60	0.17	0.47	0.48

[24] This represents the average score of all primary future teachers tested in Poland including teachers in generalist and in specialist programs; however the mathematics specialists in Poland scored higher than future teachers in Russia or in the USA in MCK (614) and in MPCK (575), and higher than generalists teachers in Poland itself (456 in MCK and 452 in MPCK). While we present the average in this table, the HLM analysis shows the difference as indicated by the large % of variance explained between programs.

[25] The USA has programs that prepare future primary teachers for generalists and specialists however there were no differences in their performance in MCK (generalists 518 and specialists 520) or MPCK (generalists 544 and specialists 545). The case is different among the secondary programs as USA future teachers trained to teach lower secondary only scored lower than their counterparts trained to also teach upper secondary in both MCK (lower secondary only 468; and lower and upper secondary 553) and MPCK (lower secondary only 471; and lower and upper secondary 542). While we present the average in this table, the HLM analysis shows the difference as indicated by the large % of variance explained between programs.

Table 6 *Summary of Regression Analyses for Variables Predicting MCK and PCK for Primary Future Teachers in TEDS-M*

	MCK								MPCK							
	Chinese Taipei		Germany		Singapore		Switzerland		Chinese Taipei		Germany		Singapore		Switzerland	
Variable	B (SE)	β	B (SE)	β	B (SE)	β	B (SE)	β	B (SE)	β	B (SE)	β	B (SE)	β	B (SE)	β
SES	1.12 (2.62)	0.01	-0.36 (3.87)	0.00	4.52 (4.05)	0.05	2.98 (2.72)	0.04	-0.79 (2.68)	-0.01	5.46 (5.94)	0.05	2.63 (3.95)	0.03	1.14 (2.28)	0.02
Age	-3.10 (0.59)	-0.08**	-0.80 (1.05)	-0.03	-1.64 (0.84)	-0.10*	-1.25 (0.67)	-0.07**	-0.86 (0.71)	-0.03	-1.47 (0.95)	-0.06+	-2.53 (0.72)	-0.17**	-2.04 (0.59)	-0.12**
Gender	-20.69 (6.46)	-0.11**	-16.63 (13.46)	-0.05	-33.22 (8.17)	-0.20**	-20.02 (5.63)	-0.11**	1.34 (4.22)	0.01	7.62 (11.44)	0.02	-10.44 (9.43)	-.06	-4.90 (6.10)	-0.03
Prior attainment	13.83 (2.76)	0.18**	18.89 (4.73)	0.19**	11.16 (3.18)	0.14**	15.50 (2.41)	0.23**	9.54 (2.29)	0.16**	22.47 (6.02)	0.20**	10.26 (3.65)	0.13**	8.74 (2.00)	0.13**
University Mathematics Geometry	98.70 (154.92)	0.21	36.37 (12.17)	0.27**	12.28 (944.25)	0.06	43.55 (11.95)	0.12**	15.52 (132.49)	0.04	26.99 (18.34)	0.18+	7.37 (1198.68)	0.04	-4.25 (12.98)	-0.01
School mathematics: functions, probability, calculus	-7.39 (85.26)	-0.02	4.57 (12.61)	0.04	-4.36 (33.73)	-0.02	-22.99 (9.61)	-0.15**	5.80 (72.01)	0.02	11.66 (19.02)	0.09	-4.67 (35.22)	-0.02	-18.43 (10.15)	-0.12*
Mathematics readings	-0.06 (8.65)	.00	-1.63 (7.57)	-0.02	-20.76 (613.94)	-0.12	-5.61 (2.63)	-0.09**	6.05 (8.90)	0.06	-4.65 (5.04)	-0.06	-18.35 (786.36)	-0.11	-7.69 (2.82)	-0.12**
program coherence	5.64 (9.95)	0.03	-8.01 (7.38)	-0.04	-8.19 (1647.31)	-0.06	25.06 (13.59)	0.16*	-8.35 (12.96)	-0.05	-8.28 (7.16)	-0.04	1.17 (2096.73)	0.01	4.30 (8.41)	0.03
mathematics as rules and procedures	-22.67 (61.92)	-0.04	-40.88 (19.30)	-0.13*	-21.10 (74.79)	-0.05	10.32 (17.75)	0.03	-5.33 (52.89)	-0.01	-31.26 (23.62)	-0.09+	-39.21 (86.65)	-0.09	-34.30 (15.03)	-0.11**
mathematics as active learning	-29.29 (62.82)	-0.04	-2.80 (14.95)	-0.02	54.28 (169.20)	0.14	-20.28 (6.43)	-0.10*	-7.81 (69.61)	-0.01	11.96 (14.50)	0.06	74.78 (215.66)	0.20	-20.77 (6.20)	-0.11**
Program SES SES_Mean5	43.39 (63.89)	0.05	65.35 (28.19)	0.10*	219.88 (8524.58)	0.16	8.35 (13.50)	0.03	25.13 (51.62)	0.04	57.76 (29.51)	0.08*	353.62 (10860)	0.26	0.61 (10.22)	0.00
R^2	.12		.25		.12		.15		.05		.22		.10		.07	

+ p < .10 *p < .05. **p < .01 (T-test > 1.96 is significant at the .05 level.)

Table 7 *Summary of Regression Analyses for Variables Predicting MCK and PCK for Secondary Future Teachers in TEDS-M*

	MCK								MPCK							
	Chinese Taipei		Germany		Singapore		Switzerland		Chinese Taipei		Germany		Singapore		Switzerland	
Variable	B (SE)	β	B (SE)	β	B (SE)	β	B (SE)	β	B (SE)	β	B (SE)	β	B (SE)	β	B (SE)	β
SES	1.75 (3.90)	0.02	10.14 (4.71)	0.10*	16.44 (3.90)	0.22**	-6.89 (4.58)	-0.13+	2.44 (5.21)	0.02	5.88 (6.59)	0.06	11.63 (5.59)	0.11**	-22.51 (6.40)	0.12**
Age	-1.82 (2.22)	-0.05	2.09 (1.03)	0.13*	-3.98 (0.72)	-0.26**	-1.78 (1.22)	-0.16+	-0.34 (3.07)	-0.01	0.72 (1.59)	0.04	-3.74 (1.02)	-0.18**	-0.66 (1.74)	-0.04
Gender	-27.65 (5.95)	-0.18**	-30.95 (10.26)	-0.15**	-22.86 (6.11)	-0.19**	-9.12 (8.31)	-0.09	-4.27 (10.75)	-0.02	-15.20 (13.58)	-0.07	-30.92 (9.73)	-0.18**	-21.70 (13.74)	-0.15+
Prior attainment	7.42 (3.94)	0.11*	23.49 (6.31)	0.24**	8.70 (2.72)	0.14**	13.00 (4.77)	0.24**	3.17 (5.35)	0.04	17.35 (6.17)	0.17**	7.56 (4.29)	0.09*	10.09 (7.06)	0.13**
University Mathematics Geometry	62.51 (12.04)	0.27**	-7.03 (12.12)	-0.04	77.97 (342.11)	0.52	-32.85 (42.54)	-0.28	87.23 (21.36)	0.30**	-4.28 (17.02)	-0.02	55.21 (337.98)	0.27	-38.40 (73.23)	-0.23
School mathematics: functions, probability, calculus	19.44 (11.36)	0.09*	54.29 (15.13)	0.37**	-165.49 (740.58)	-0.82	22.03 (57.86)	0.18	-5.71 (21.47)	-0.02	40.45 (19.46)	0.27*	-155.65 (717.77)	-0.56	44.13 (103.18)	0.25
Mathematics readings	-6.86 (4.14)	-0.08*	-12.23 (6.51)	-0.06*	183.05 (733.78)	0.48	-8.64 (47.12)	-0.15	2.37 (8.09)	0.02	-8.20 (8.25)	-0.04	168.27 (706.66)	0.32	-21.99 (86.17)	-0.27
program coherence	1.66 (8.53)	0.01	-2.87 (6.60)	-0.01	-16.63 (572.19)	-0.05	8.03 (26.18)	0.15	-7.05 (11.98)	-0.04	-10.59 (11.50)	-0.05	-11.07 (565.24)	-0.02	13.05 (47.98)	0.17
mathematics as rules and procedures	-21.25 (18.16)	-0.06	-107.75 (35.28)	-0.15**	-17.82 (150.53)	-0.02	-96.74 (191.98)	-0.53	-33.30 (26.15)	-0.07	-111.17 (37.18)	-0.15**	-31.12 (162.97)	-0.02	-78.83 (329.84)	-0.30
mathematics as active learning	10.00 (16.56)	0.03	-5.46 (15.15)	-0.03	-55.01 (53.93)	-0.13	-35.09 (43.43)	-0.29	28.08 (20.64)	0.08+	-18.24 (21.23)	-0.09	-34.15 (65.33)	-0.06	-12.26 (69.95)	-0.07
Program SES SES_Mean5	11.93 (17.19)	0.03	0.23 (16.68)	0.00	-122.28 (992.57)	-0.22	8.19 (68.18)	0.03	71.91 (29.76)	0.15**	2.75 (16.97)	0.01	-91.04 (960.52)	-0.12	66.49 (132.52)	0.19
R^2	.16		.39		.30		.19		.12		.18		.10		.16	

+ p < .10 *p < .05. **p < .01 (T-test > 1.96 is significant at the .05 level.)

Table 8 HLM Model of Future Primary and Secondary Teachers' Mathematics Knowledge for Teaching Given Background and Program Characteristics[26]

VARIABLES AND LEVELS	PRIMARY PROGRAMS						SECONDARY PROGRAMS					
	Poland		Russian Federation		United States		Poland		Russian Federation		United States	
	Coefficient		Coefficient		Coefficient		Coefficient		Coefficient		Coefficient	
	MCK	MPCK	MCK	MPCK	MCK	MPCK	MCK	MPCK	MCK	MPCK	MCK	MPCK
Future Teacher characteristics												
SES*[SES]	0.03+	0.03	0.01	-0.01	0.05+	0.07**	0.04	0.05	-0.01	0.00	0.04	0.06
Age [MFA001]	-0.04+	-0.11***	0.03+	0.03	-0.06	-0.09**	-0.13*	-0.10+	-0.01	-0.01	-0.10*	-0.08
Gender [MFA002]1=F; 0=M	-0.10***	-0.04*	-0.06*	-0.00	-0.15***	-0.04*	-0.16**	-0.14**	0.04*	0.05*	-0.26***	-0.14*
Prior attainment: [MFA009]	0.13***	0.12***	0.07***	0.08***	0.17***	0.15***	0.17*	0.08	0.05**	0.06*	0.18***	0.11*
Teacher Education Program Characteristics												
Opportunities to Learn												
University level mathematics: Geometry [MFB1GEOM]	0.12*	0.10*	0.00	0.02	0.02	-0.02	0.14+	0.08	0.13*	0.16***	0.28***	0.15*
School level mathematics: function, probability and calculus [MFB2SLMF]	0.33***	0.19**	0.24**	0.03	0.01	0.02	-0.02	-0.10	0.03	-0.03	0.09	0.13*
Reading research on teaching and mathematics [MFB5READ]	-0.06	-0.03	0.04	0.07	0.02	0.01	-0.09	-0.11	0.20+	0.26**	-0.06	-0.04
Program coherence [MFB15COH]	0.01	-0.01	0.30**	0.26**	-0.05	-0.04	0.03	0.05	0.17	0.07	-0.09*	-0.02
Philosophy (views)												
View of mathematics is a collection of rules and procedures [MFD1RULE]	-0.11+	-0.09+	-0.28**	-0.24*	-0.08*	-0.09*	-0.24	-0.26*	-0.34***	-0.11+	-0.19***	-0.24***
View that mathematics is better learned through active learning [MFD2ACTV]	0.06	0.09*	-0.08	0.03	0.09*	0.03	-0.02	-0.00	-0.01	-0.00	-0.00	-0.04
Average SES for each program (aggregated from future teachers SES) [SES]	0.07+	0.03	0.01	0.02	0.21***	0.12***	0.12	-0.00	0.02	0.05	0.04	0.02
% of variance explained within programs (Level 1)	5%	3%	0%	0%	5%	5%	6%	1%	0%	1%	12%	5%
% of variance explained between programs (Level 2)	85%	80%	36%	18%	82%	99%	50%	35%	31%	31%	84%	86%
ICC	60%	39%	40%	37%	16%	5%	30%	24%	50%	26%	38%	17%

+ $p \leq .10$; * $p \leq .05$; ** $p \leq .01$; *** $p \leq .001$

[26] The coefficients have been standardized.

Appendix

Table A1 Variables in the Model and Model Specification: Future Teacher Characteristics and Program Characteristics

Variables in the Model

Future Teachers Characteristics

Socioeconomic status [SES]
A scale was created by averaging the possessions in the parents or guardians home variables: using principal components analysis to create a scale with the home possessions scale, number of books at home, father's highest level of education and mother's highest level of education, so we get a proxy measure of SES

Age [MFA001] - As reported

Gender [MFA002] - 1=female, 0=male

Prior attainment [MFA009]
Level of grades in secondary school (1=below average for year level; 5=Always at top of year level)

Program Characteristics

Opportunities to learn

Tertiary Level Mathematics – Geometry [MFB1GEOM]
A. Foundations of Geometry or Axiomatic Geometry (e.g., Euclidean axioms)
B. Analytic/Coordinate Geometry (e.g., equations of lines, curves, conic sections, rigid transformations or isometrics)
C. Non-Euclidean Geometry (e.g., geometry on a sphere)
D. Differential Geometry (e.g., sets that are manifolds, curvature of curves, and surfaces)
Calculating the mean value for each program eliminating the cases where more than 90% of the FT's values were missing for each program, so we get the OTL - Tertiary Level Math - Geometry aggregated at the program level (range 0-4)

School Level Mathematics - Function Probability Calculus [MFB2SLMF]
D. Functions, Relations, and Equations (e.g., algebra, trigonometry, analytic geometry)
E. Data Representation, Probability, and Statistics
F. Calculus (e.g., infinite processes, change, differentiation, integration)
G. Validation, Structuring, and Abstracting (e.g., Boolean algebra, mathematical induction, logical connectives, sets, groups, fields, linear space, isomorphism, homomorphism)
Calculating the mean value for each program eliminating the cases where more than 90% of the FT's values were missing for each program, so we get the OTL - School Level Math - Functions Probability Calculus aggregated at the program level (range 0-4).

Mathematics Education Pedagogy - Class Reading [MFB5READ]
H. Read about research on mathematics
I. Read about research on mathematics education
J. Read about research on teaching and learning
K. Analyze examples of teaching (e.g., film, video, transcript of lesson)
Scaled with mean =10 representing the neutral point.

Program Coherence [MFB15COH]
A. Each stage of the program seemed to be planned to meet the main needs I had at that stage of my preparation.
B. Later <courses> in the program built on what was taught in earlier <courses> in the program.
C. The program was organized in a way that covered what I needed to learn to become an effective teacher.
D. The <courses> seemed to follow a logical sequence of development in terms of content and topics.
E. Each of my <courses> was clearly designed to prepare me to meet a common set of explicit standard expectations for beginning teachers.
F. There were clear links between most of the <courses> in my teacher education program.
Scaled with mean =10 representing the neutral point.

Table A1 (continued) Variables in the Model: Future Teacher Characteristics and Program Characteristics

Program Philosophy (views)
Rules and Procedures [MFD1RULE] 1. Mathematics is a collection of rules and procedures that prescribe how to solve a problem. 2. Mathematics involves the remembering and application of definitions, formulas, mathematical facts and procedures. 3. When solving mathematical tasks you need to know the correct procedure else you would be lost. 4. Fundamental to mathematics is its logical rigor and preciseness. 5. To do mathematics requires much practice, correct application of routines, and problem solving strategies. 6. Mathematics means learning, remembering and applying. Calculating the mean value for each program eliminating the cases where more than 90% of the FT's values were missing for each program, so we get the BELIEFS - Rules and Procedures aggregated at the program level Scaled with mean =10 representing the neutral point.
Active Learning [MFD2ACTV] 1. In addition to getting a right answer in mathematics, it is important to understand why the answer is correct. 2. Teachers should allow pupils to figure out their own ways to solve mathematical problems. 3. Time used to investigate why a solution to a mathematical problem works is time well spent. 4. Pupils can figure out a way to solve mathematical problems without a teacher's help. 5. Teachers should encourage pupils to find their own solutions to mathematical problems even if they are inefficient. 6. It is helpful for pupils to discuss different ways to solve particular problems. Calculating the mean value for each program eliminating the cases where more than 90% of the FT's values were missing for each program, so we get the BELIEFS - Active Learning aggregated at the program level Scaled with mean =10 representing the neutral point.
Program's SES [SES] Calculating the mean value for each program eliminating the cases where more than 90% of the FT's values were missing for each program, so we get the SES measure aggregated at the program level (based on FTQ)

Model Specification (HLM)

Summary of the model specified (in equation format)

STEP 1

Full Unconditional Model (Level 1 & 2)

$$Y_{ij} = G_{00} + u_{0j} + r_{ij}, \text{ where } u_{0j} \sim N(0, \tau_{00}) \text{ and } r_{ij} \sim N(0, \sigma^2)$$

The ICC = $(\tau_{00}) / (\tau_{00} + \sigma^2)$

STEP 2

Level-1 Model

$$Y_{ij} = B_{0j} + B_{1j}(SES)_{ij} + B_{2j}(MFA001)_{ij} + B_{3j}(MFA002)_{ij} + B_{4j}(MFA009)_{ij} + r_{ij}, \text{ where } r_{ij} \sim N(0, \sigma^2)$$

Level-2 Model

$$\begin{aligned} B_{0j} = G_{00} &+ G_{01}(MFB1GEOM)_j + G_{02}(MFB2SLMF)_j + G_{03}(MFB5READ)_j + G_{04}(MFB15COH)_j \\ &+ G_{05}(MFD1RULE)_j + G_{06}(MFD2ACTV)_j + G_{07}(MIG001A)_j + G_{08}(MIC002B)_j \\ &+ G_{09}(MID005B)_j + G_{010}(MID008B)_j + G_{011}(MIE002AB)_j + G_{012}(MIE002CD)_j \\ &+ G_{013}(MIG001C2)_j + G_{014}(MSES)_j + u_{0j}, \text{ where } u_{0j} \sim N(0, \tau_{00}) \end{aligned}$$

$$B_{1j} = G_{10}$$
$$B_{2j} = G_{20}$$
$$B_{3j} = G_{30}$$
$$B_{4j} = G_{40}$$

Appendix 1

Table A1 Results of the Mathematics Content Knowledge Assessment for Future Primary Teachers

Program Group	Country	Sample Size	Valid Data (N)	Percent Missing (Weighted)	Percent at or above Anchor Point 1 (SE)		Percent at or above Anchor Point 2 (SE)		Scaled Score: Mean (SE)	
Group 1. Lower Primary (Grade 4 Maximum)	Germany	935	907	2.4	86.4	(1.3)	43.9	(2.1)	501	(3)
	Poland [a]	1812	1799	0.9	67.9	(1.3)	16.8	(1.2)	456	(2)
	Russian Federation [b]	2266	2260	0.2	89.7	(2.3)	57.3	(4.6)	536	(10)
	Switzerland [c]	121	121	0.0	90.5	(2.7)	44.2	(5.4)	512	(6)
Group 2. Primary (Grade 6 Maximum)	Chinese Taipei	923	923	0.0	99.4	(0.3)	93.2	(1.4)	623	(4)
	Singapore	263	262	0.4	100.0		82.5	(2.3)	586	(4)
	Switzerland	815	815	0.0	97.2	(0.6)	70.6	(1.4)	548	(2)
	† USA [d]	1310	951	28.6	92.9	(1.2)	50.0	(3.2)	518	(5)
Group 4. Primary Specialists	Germany	97	97	0.0	96.0	(2.1)	71.7	(7.0)	555	(8)
	Poland [a]	300	300	0.0	97.9	(1.0)	91.0	(1.6)	614	(5)
	Singapore	117	117	0.0	98.3	(1.2)	87.3	(2.8)	600	(8)
	† USA [d]	191	132	33.2	94.9	(1.7)	48.1	(6.5)	520	(7)

The dagger symbol (†) is used to alert readers to situations where data were available from less than 85% of respondents.
The shaded areas identify data that, for reasons explained in the annotations, can be compared with data from other countries with caution.
The solid vertical lines on the chart show the two Anchor Points (431 and 516). Country Annotations for Primary MCK

a. Poland: Reduced coverage: institutions with consecutive programs only were not covered. Combined participation rate between 60 and 75%.
b. Russian Federation: Reduced coverage: secondary pedagogical institutions were excluded.
c. Switzerland: Reduced coverage: the population covered includes only institutions where German is the primary language of use and instruction.
d. USA: Reduced coverage: public institutions only. Combined participation rate between 60% and 75%. An exception was made to accept data from two institutions because, in each case, one additional participant would have brought the response rate above the 50% threshold. Although the participation rate for the complete sample meets the required standard, the data contain records that were completed using a telephone interview, when circumstances did not allow administration of the full questionnaire. Of the 1501 recorded as participants, the full questionnaire was administered to 1185. Bias may arise in the data because of the number of individuals who were not administered the full questionnaire.

Table A2 Results of the Mathematics Pedagogy Knowledge Assessment for Future Primary Teachers[13]

Program Group	Country	Sample Size	Valid Data (N)	Percent Missing (Weighted)	Percent at or above Anchor Point (SE)	Scaled Score: Mean (SE)
Group 1. Lower Primary (Grade 4 Maximum)	Germany	935	907	2.4	25.9 (2.0)	491 (5)
	Poland [a]	1812	1799	0.9	11.9 (1.3)	452 (2)
	Russian Federation [b]	2266	2260	0.2	31.6 (4.1)	512 (8)
	Switzerland [c]	121	121	0.0	31.6 (4.2)	519 (6)
Group 2. Primary (Grade 6 Maximum)	Chinese Taipei	923	923	0.0	77.0 (1.3)	592 (2)
	Singapore	263	262	0.4	74.9 (2.5)	588 (4)
	Switzerland	815	815	0.0	44.0 (1.5)	539 (2)
	† USA [d]	1310	951	28.6	47.6 (1.7)	544 (3)
Group 4. Primary Specialists	Germany	97	97	0.0	59.6 (3.4)	552 (7)
	Poland [a]	300	300	0.0	67.3 (2.3)	575 (4)
	Singapore	117	117	0.0	81.1 (3.9)	604 (7)
	† USA [d]	191	132	33.2	41.4 (6.3)	545 (6)

The dagger symbol (†) is used to alert readers to situations where data were available from less than 85% of respondents.
The shaded areas identify data that, for reasons explained in the annotations, can be compared with data from other countries with caution.
The solid vertical lines on the chart show the Anchor Points (544). Country Annotations for Primary MCK
a. Poland: Reduced coverage: institutions with consecutive programs only were not covered. Combined participation rate between 60 and 75%.
b. Russian Federation: Reduced coverage: secondary pedagogical institutions were excluded.
c. Switzerland: Reduced coverage: the population covered includes only institutions where German is the primary language of use and instruction.
d. USA: Reduced coverage: public institutions only. Combined participation rate between 60% and 75%. An exception was made to accept data from two institutions because, in each case, one additional participant would have brought the response rate above the 50% threshold. Although the participation rate for the complete sample meets the required standard, the data contain records that were completed using a telephone interview, when circumstances did not allow administration of the full questionnaire. Of the 1501 recorded as participants, the full questionnaire was administered to 1185. Bias may arise in the data because of the number of individuals who were not administered the full questionnaire.

Table A3 Results of the Mathematics Content Knowledge Assessment for Future Secondary Teachers

Program Group	Country	Sample Size	Valid (N)	Percent Missing (Weighted)	Percent at or above AP1490 (SE)		Percent at or above AP 2 559 (SE)		Scale Score Mean (SE)	
5. Lower Secondary (to Grade 10 Max.)	**Germany**	408	406	0.3	53.5	(3.4)	12.6	(2.2)	483	(5)
	Poland [c]	158	158	0.0	75.6	(3.5)	34.7	(3.2)	529	(4)
	Singapore	142	142	0.0	86.9	(3.1)	36.6	(4.3)	544	(4)
	Switzerland [d]	141	141	0.0	79.7	(3.4)	26.7	(3.2)	531	(4)
	† **USA** [f]	169	121	32.7	33.5	(2.2)	2.1	(1.3)	468	(4)
6. Lower & Upper Secondary (to Grade 11 & above)	**Chinese Taipei**	365	365	0.0	98.6	(0.8)	95.6	(1.0)	667	(4)
	Germany	363	362	0.1	93.4	(1.5)	62.1	(2.9)	585	(4)
	Poland	140	139	0.8	85.7	(2.6)	35.7	(2.7)	549	(4)
	Russian Federation [h]	2141	2139	0.1	88.8	(1.7)	61.1	(4.3)	594	(13)
	Singapore	251	251	0.0	97.6	(1.0)	62.9	(2.6)	587	(4)
	† **USA** [f]	438	354.0	21.3	87.1	(2.0)	44.5	(3.9)	553	(5)

(†) Data were available from less than 85% of respondents. The shaded areas identify data that, for reasons explained in the annotations, can be compared with data from other countries only with caution. MCK AP1 (Anchor Point 1=490), MCK AP2 (Anchor Point 2=559); PCK AP (Anchor Point=509).

c. Poland: Reduced coverage: institutions with consecutive programs only were not covered. Combined participation rate between 60 and 75%.
d. Switzerland: Reduced coverage: includes only institutions where German is the primary language of use and instruction
f. USA: Reduced coverage: public institutions only. Combined participation rate between 60% and 75%. An exception was made to accept data from one institution because one additional participant would have brought the response rate above the 50% threshold. Although the participation rate for the complete sample meets the required standards, the data contain records that were completed using a telephone interview, when circumstances did not allow administration of the full questionnaire. Of the 607 recorded as participants, the full questionnaire was administered to 502. Bias may arise in the data because significant numbers of individuals were not administered the full questionnaire.
h. Russian Federation: An unknown number of those surveyed had previously qualified to become primary teachers.

Table A4 Results of the Mathematics Pedagogy Knowledge Assessment for Future Secondary Teachers

Program Group	Country	Sample Size	Valid (N)	Percent Missing (Weighted)	Percent at or above AP 509 (SE)		Scale Score Mean (SE)	
5. Lower Secondary (to Grade 10 Max.)	**Germany**	408	406	0.3	52.5	(4.6)	515	(6)
	Poland [c]	158	158	0.0	49.7	(3.1)	520	(5)
	Singapore	142	142	0.0	65.9	(4.2)	539	(6)
	Switzerland [d]	141	141	0.0	70.9	(3.8)	549	(6)
	† USA [f]	169	121	32.7	16.7	(3.1)	471	(4)
6. Lower & Upper Secondary (to Grade 11 & above)	**Chinese Taipei**	365	365	0.0	93.3	(1.5)	649	(5)
	Germany	363	362	0.1	80.3	(2.7)	586	(7)
	Poland	140	139	0.8	62.2	(4.7)	528	(6)
	Russian Federation [h]	2141	2139	0.1	71.0	(3.1)	566	(10)
	Singapore	251	251	0.0	75.3	(3.1)	562	(6)
	† USA [f]	438	354.0	21.3	61.0	(3.0)	542	(6)

(†) Data were available from less than 85% of respondents. The shaded areas identify data that, for reasons explained in the annotations, can be compared with data from other countries only with caution. MCK AP1 (Anchor Point 1=490), MCK AP2 (Anchor Point 2=559); PCK AP (Anchor Point=509).

c. Poland: Reduced coverage: institutions with consecutive programs only were not covered. Combined participation rate between 60 and 75%.
d. Switzerland: Reduced coverage: includes only institutions where German is the primary language of use and instruction
f. USA: Reduced coverage: public institutions only. Combined participation rate between 60% and 75%. An exception was made to accept data from one institution because one additional participant would have brought the response rate above the 50% threshold. Although the participation rate for the complete sample meets the required standards, the data contain records that were completed using a telephone interview, when circumstances did not allow administration of the full questionnaire. Of the 607 recorded as participants, the full questionnaire was administered to 502. Bias may arise in the data because significant numbers of individuals were not administered the full questionnaire.
h. Russian Federation: An unknown number of those surveyed had previously qualified to become primary teachers.