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Analysis of school-level factors contributing to spatial inequality in academic achievement in Italy

Patrizia Falzetti - INVALSI

Chiara Sacco - INVALSI

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Abstract

A hot research topic in education is the study of the territorial differences. Focusing on the Italian educational system, large-scale international and national survey assessments have been widely used to investigate the educational disparities across regions. The aim of this work is to study the spatial disparities in the relationship between academic achievements and some school-level factors moving beyond regional administrative confines. This study exploits the standardized test administered by INVALSI for the school years 2017-2018 focusing on 8th grade students. Crucial to our contribution is the use of the geographically weighted regression and the k-mean clustering, which allows to study the spatial variability of the impact of the school-level factors on the academic achievement and to gather schools in new spatial clusters. The identification of school clusters, homogeneous with respect to the factors' impact on school performance, reflects the complexity of the Italian reality and highlights the necessity to differentiate the support for schools on the basis of their specific need.

Parole chiave: Disuguaglianza; Analisi spaziale; Large-scale Assessment; Scuola secondaria di primo grado; Regressione geografica pesata

Keywords: Inequality; Spatial Analysis; Large-scale Assessment; Lower Secondary School; Geographically Weighted Regression

Introduction

Large-scale national and international survey assessments, such as Programme for International Student Assessment (PISA), Trends in International Mathematics and Science Study (TIMSS), and INVALSI national test, have been widely used for decades to measure what students know and can do in order to provide an image of the school system's status. Educational equality means that the school system is able to offer to each students the same opportunities to achieve the best possible outcome, in terms of academic skills, regardless of gender, ethnic origin, socio-economic status and where one lives. A priority for policies makers is to guarantee equity in education. Recently, the study of territorial difference in academic performance distribution is of particular interest for addressing policy decisions. Agasisti and Cordero-Ferrara (2013) proposed a multilevel analysis applied to PISA 2006 data to study educational disparities across regions in Italy and Spain, whereas Matteucci and Mignani (2014), using a multilevel approach, verified the existence of regional and macro-area differences in reading competencies of Italian students starting from an analysis of PISA 2009 data. Hippe et al. (2018) exploited the regional distribution of skills in Italy and Spain, studying the extent of the regional inequalities in PISA 2015 using descriptive statistics and estimating several regression models. Costanzo and Desimoni (2017) explored inequalities in education across Italian geographic areas using a quantile regression approach applied to primary school data from INVALSI large-scale assessments. The annual INVALSI technical brief (2019) showed the regional distribution of skills in Italy for primary and secondary school and analysed the longitudinal trends of the regional performance of the last two years.

The presence of the well-known regional inequalities in education in Italy points out the need to better understand the regional disparieties and the connected policy implications since it has been demonstrated that the economic development of a region is highly correlated to the level of human capital (Gennaioli et al, 2012). We propose to analyse territorial difference using the geo-coded school information, that provides a new prospective not only visualizing summative maps but also offering the possibility of studying new spatial patterns of schools' performance. This work exploits the standardized test administered by INVALSI for the school years 2018-2019 focusing on 8th grade students, for which the test is compulsory for admission to the state final exam. The use of school level census data and geo-referenced data allows to analyse the spatial variability of academic achievements among schools in Italy.

The aim of the study is to examine the distribution of academic achievements at school level identifying for each school the key school-predictors of spatial inequalities and to investigate the extent of the spatial disparities in the relationship between the academic achievement and the school-level predictors. In order to analyse the geographic dimension of the relationship between academic achievement and predictors, we employed two statistical models: the ordinary least square (OLS) regression and the geographically weighted regression (GWR). The traditional OLS regression model studies the global relationship between the school performance and the predictors, whereas the GWR examines the spatially varying relationship. GWR allows to determine whether the underlying data generating process shows spatial heterogeneities or local deviations from the global regression model estimating local regression coefficients (Fotheringham et al., 2003; Lloyd, 2010). GWR is a powerful exploratory method in spatial data analysis, widely used in epidemiology; few applications of GWR in the educational field have been proposed (Fotheringham et al., 2001; Harris et al., 2010; Naidoo et al., 2014). The novelty of the approach is to apply k-means clustering on the local regression coefficients to identify clusters of school with distinct regression patterns. Cluster identification allows to better understand how differentiate supports for schools on the basis of their unique contextual needs.

Data

The National Evaluation Institute for the School System (INVALSI), annually, carries out standardized tests to assess the performance of all Italian students at the end of the second and the fifth years of primary school, at the end of lower secondary school, and at the end of the second year of higher secondary school. In order to study the territorial difference in the performances in Italian Language, we exploited the standardized test administered by the INVALSI at the end of lower secondary school in the school year 2018-2019. INVALSI standard test is compulsory at the end of the lower secondary school for all students to be admitted to the state final exam. Furthermore, at the end of the lower secondary school the INVALSI test administration mode is computer-based. Computer-based administration and the test compulsoriness provided a consistent and cleaner dataset and allowed to minimize cheating and, thus, to avoid cheating correction procedure (Quintano et al., 2009; Longobardi et al., 2018). INVALSI collected data on socio-demographic variables through a questionnaire, compiled by students at the end of the test; the school's secretarial offices provide further information about students, school and classes characteristics. The geospatial location of each school is extracted from the school address provided by the school secretarial office.

In the school year 2018-2019, the INVALSI test has been administered to a population of 542.689 students in 5.761 schools. In this study, we considered the data aggregated at school-level and we focused on schools with: at least 20 students, a percentage of students' participation higher than 80%, geospatial location available. The number of schools analyzed in this study is 5.520.

Inequalities in the distribution of school mean performances[1] were estimated using a set of socioeconomic and demographic variables: school socio-economic status indicator (ESCS, Campodifiori et al.), mean WLE score at INVALSI test at the end of primary school in the s.y. 2015-2016 (called WLE in G05), number of classes, number of school buildings, percentage of foreign students, percentage of repeating students, percentage of females.

Methods

To assess the geographic dimension of the association between schools' performance and ESCS, WLE in G05, number of classes, number of school buildings, percentage of foreign students, percentage of repeating students and percentage of females, this study followed a two steps procedure. First, both linear regression model and geographically regression model were estimated to study the association at global and local level, respectively. All the variables included in both models have been standardized with mean 0 and variance 1.

An OLS model can be expressed as

$$y_i = \beta_0 + \sum_k \beta_k x_{ik} + \varepsilon_i \qquad [1]$$

where y_i is the dependent variable for i-th observation, β_0 is the intercept, β_k is the coefficient for the predictor x_k, x_{ik} is the k-th predictor for i-th observation and ε_i is the error term. GWR allows the relationship between the dependent variable and the predictors to vary geographically considering locally weighted regression coefficients. In the framework of GWR model (Brunsdon et al., 1998), the relationship between y_i and the predictors can be written as

$$y_i = \beta_0(u_i, v_i) + \sum_k \beta_k(u_i, v_i) x_{ik} + \varepsilon_i \qquad [2]$$

where (u_i, v_i) denotes the geographical coordinates of the i-th observation. Thus, $\beta_0(u_i, v_i)$ represents the intercept at the location i with coordinates (u_i, v_i) and $\beta_k(u_i, v_i)$ denotes the coefficient for the predictor x_k at the location i with coordinates (u_i, v_i). Instead of estimating one single regression, the GWR model generates separate regressions, one for each observation. In this way, GWR allows to examine the spatial variation in the relationship between the dependent variable and the predictors. The GWR model calibrates

[1] The students' performances at Italian INVALSI test are measured using the Weighted Likelihood Estimation (WLE) estimated by the Rasch model.

separate regressions for each observation assuming that observations are weighted on the base of their proximity to *i*-th (closer observations have greater influence on the estimation of regression parameters of *i*-th observations than remote observations). The decrease in the weight of each observation with distance to the point of interest is determined by a kernel function. The key parameters of the kernel function are the kernel shape and the bandwidth size. In this study, we assumed a Gaussian adaptive kernel, that allows the bandwidth to vary on the base of the density of observation points. The optimal number of the nearest neighbour N included in the estimation, the bandwidth, has been computed through a calibration process based on the minimization of the Akaike Information Criteria (Fotheringham *et al.*, 2003). The spatial analyses have been performed using the R package *spdep* (Bivand, Wong, 2018; Bivand et al., 2013) e *GWmodel* (Gollini et al., 2013; Binbin et al., 2014).

In the second step of the procedure, k-means clustering method has been applied on the matrix of the spatial coefficients in order to divide schools into homogeneous regions based on local regression patterns. K-means clustering is one of the most popular unsupervised machine learning algorithms, with the objective to group similar data points together and discover underlying patterns. To achieve this objective, k-means looks for a fixed number (k) of clusters in a dataset. Determining the optimal number k of clusters in a data set is a fundamental issue and, in literature, a wide variety of indices have been proposed. For the identification of the number of clusters the package *NbClust* package (Charrad et al., 2014) has been used. *NbClust* package provides 30 indices for determining the optimal number of clusters and proposes to user the best clustering scheme from different results obtained by varying all combinations of number of cluster.

Results

To examine the geographical patterns in the relationship between schools' performance and ESCS, WLE in G05, number of classes, number of school buildings, percentage of foreign students, percentage of repeating students and percentage of females, the OLS and GWR models were estimated. In the framework of the OLS, three models have been estimated: Model 1 with only the socio-demographic variables, Model 2 and Model 3 with the socio-demographic variables and a variable to account for the geographic location of the school. In Model 2 the geographic location is included as a variable with five different levels, each level is a geographic macro-area, whereas in Model 3 the geographic location of the school is represented by the region. Table 1 reports different measures of goodness of fit for all four models (OLS: Model 1, Model 2, Model 3; GWR: Model 4). The GWR fits better the data as it has higher adjusted R^2 and lower AICc. In correspondence of the

Model 1, the adjusted R^2 is equal to 0,60, which indicates that about the 60% of the total variance of the dependent variable is explained by the model, whereas for the GWR the adjusted R^2 is equal to 0,70. Adding the geographic variables in the OLS model leads to an increment of the R^2 measures and a reduction in terms of AIC; in other words, taking into account for the geographic location allows to explain a greater variability of the performance of the school. Differently to the GWR, that accounts for the geographic location of the schools and assesses the spatial heterogeneity of the coefficients, the inclusion in the OLS of the geographic variable as fixed effect (Model 2, Model 3) allows only to differentiate the interpretation of the intercept, as shown in Table 2.

Table 1. Goodness-of-fit model comparison

	OLS Model			**GWR Model**
	Model 1	**Model 2**	**Model 3**	**Model 4**
R-Squared	0,60	0,66	0,68	0,71
Adjusted R-Squared	0,60	0,66	0,68	0,70
AIC	10593,93	9669,65	9345,42	8975,92
AICc	10593,96	9669,72	9537,29	9255,74

Table 2 summarizes the coefficients estimates of OLS models that pictures the relationships across the entire country. All the predictors included in Model 1 are significant; only the number of buildings and the percentage of repeating students have a negative impact on the mean school performance. For Model 1, the multicollinearity has been verified using the variance inflation factor (VIF): the VIF is close to one for each predictor, suggesting that multicollinearity within the model is small. The number of the classes is the only variable that results statistically significant in Model 1 and not statistically significant in correspondence of Model 2 and Model 3.

Figure 1a shows the residuals of the Model 1 and suggests the presence of spatially dependence in the residuals: the Moran's I is statistically different from zero (I = 0,085, p < $2{,}2E\text{-}16$) and confirms the spatial autocorrelation among model residuals. For Model 2 and Model 3 we observe a slightly reduction of the Moran's I, I = 0,031 (p < $2{,}2E\text{-}16$) and I = 0,013 (p < $2{,}2E\text{-}16$), respectively. The GWR residuals exhibits a reduction of the spatial pattern (Figure 1b); the Moran I (I = 0,007, p < 5,39E-07) suggests a substantial reduction although not a total elimination of the spatial autocorrelation among model residuals.

Table 2. Global regression models' results

	Coefficients					
	Model 1		Model 2		Model 3	
Intercept	0,000	***	0,331	***	0,641	***
WLE in G05	0,326	***	0,258	***	0,228	***
ESCS	0,521	**	0,421	***	0,429	***
Number of classes	0,023	**	-0,006		0,004	
Number of buildings	-0,027	***	0,030	***	0,025	**
Female (%)	0,059	***	0,057	***	0,057	***
Foreign (%)	0,072	***	-0,091	***	-0,098	***
Repeating (%)	-0,068	***	-0,102	***	-0,110	***
Areageo: Northeast			-0,035			
Areageo: Center			-0,285	***		
Areageo: South			-0,646	***		
Areageo: South and islands			-0,835	***		
Region:Piedmont					-0,384	**
Region:Liguria					-0,538	***
Region:Lombardy					-0,233	
Region:Veneto					-0,292	*
Region:Friuli-Venezia Giulia					-0,244	
Region:Emilia-Romagna					-0,417	**
Region:Tuscany					-0,522	***
Region:Umbria					-0,605	***
Region:Marche					-0,392	**
Region:Lazio					-0,683	***
Region:Abruzzo					-0,734	***
Region:Molise					-0,797	***
Region:Campania					-1,111	***
Region:Apulia					-0,853	***
Region:Basilicata					-0,970	***
Region:Calabria					-1,506	***
Region:Sicily					-1,180	***
Region:Sardinia					-0,713	***
Region:Prov Aut Bolzano (l. it.)					-0,563	**
Region:Prov Aut Trento (l. it.)					-0,277	

Figure 1. Residuals of a) the OLS model (Model 1) and b) the GWR model (Model 4).

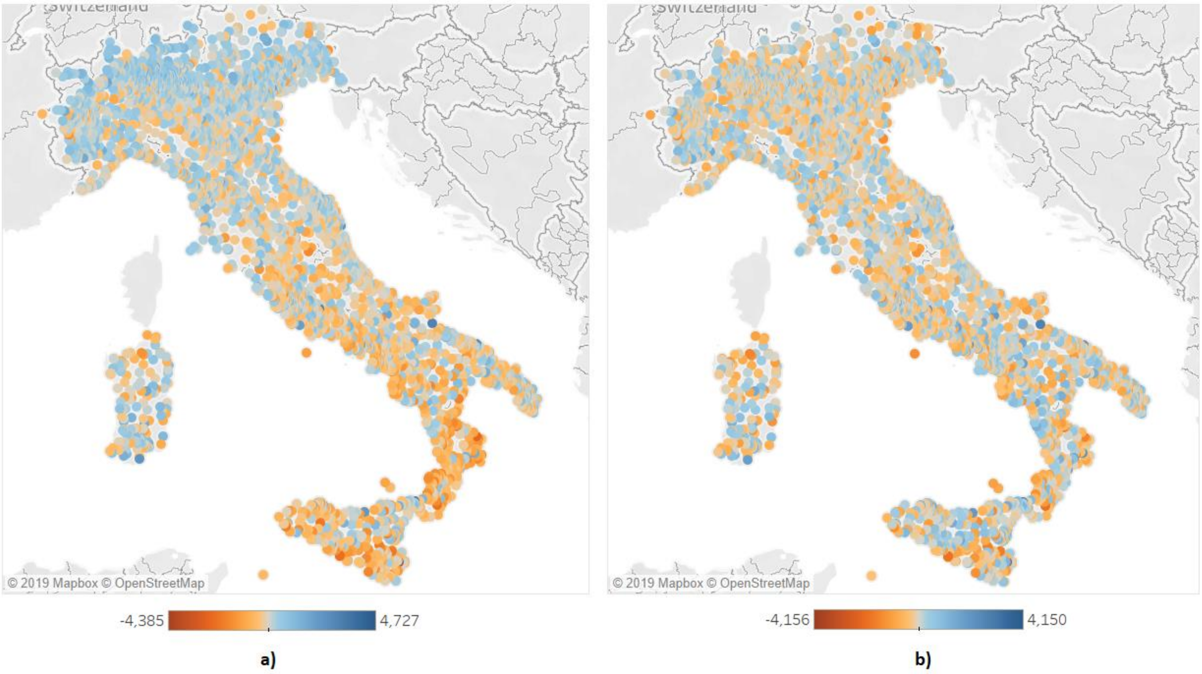


Table 2. Results for geographically weighted regression

	Coefficient Range						P-value< 0.5
	Min	**1st Quantile**	**Mean**	**Median**	**3rd Quantile**	**Max**	**% Significant coefficients**
Intercept	-1,024	-0,264	0,008	0,136	0,320	0,684	85,525
WLE in G05	0,073	0,192	0,248	0,257	0,305	0,471	99,656
ESCS	0,171	0,295	0,404	0,402	0,511	0,627	100,000
Number of classes	-0,104	-0,029	0,008	0,016	0,053	0,187	13,587
Number of buildings	-0,180	-0,032	0,016	0,014	0,048	0,175	17,572
Female (%)	-0,070	0,033	0,057	0,055	0,084	0,208	34,837
Foreign (%)	-0,463	-0,177	-0,126	-0,125	-0,062	0,397	49,692
Repeating (%)	-0,313	-0,133	-0,090	-0,089	-0,037	0,112	38,949
Local R-Squared	0,253	0,626	0,686	0,703	0,768	0,947	

Another measure of goodness of fit of the GWR model is provided by the local R^2 that ranges from 0,25 to 0,95, with the 75% of schools having a value of 0,63 or higher (Table 3).

Table 3 shows the estimated coefficients of the GWR model. The WLE obtained in G05 and the ESCS result to be consistently positively associated with the mean school performance, whereas the other indicators have both positive and negative values in different locations. Small interquartile range is observed in correspondence of the percentage of females, the number of buildings, the number of classes and the percentage of repeating students. These variables result to be not significant in the majority of the locations. For the WLE in G05, the ESCS, the percentage of foreign students and number of buildings, the interquartile range of the GWR coefficients falls below the magnitude of the OLS coefficients.

Figure 2. Coefficient estimates of the GWR model: a) intercept; b) WLE in G05; c) ESCS; d) number of classes; e) number of buildings; f) percentage of females; g) percentage of foreign students; h) percentage of repeating students.

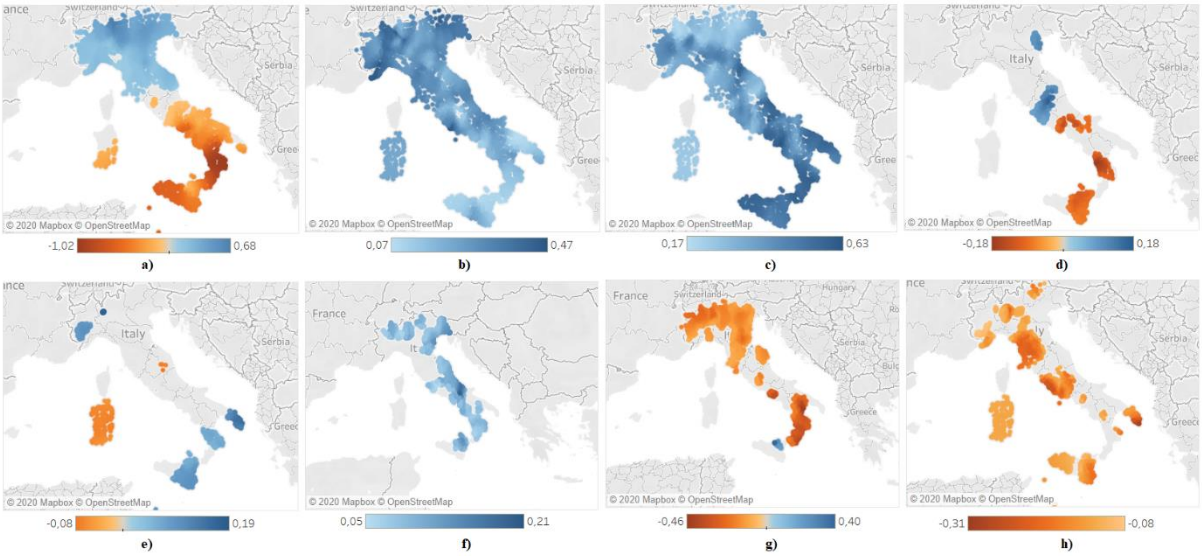


Figure 2 shows the spatial patterns of the intercept and the estimated coefficients of the 7 predictors. Only the significant coefficients are represented in the map. The WLE obtained in 5th grade has a negative impact on the mean performance in South Italy, in the north of Marche, in Modena and in Brescia province. On the other hand, a positive effect of WLE is observed in North Italy, in the metropolitan area of Rome and in the coastal area of Abruzzo and Molise. The socio-economic indicator has a high positive impact in South Italy and in few areas of Center and North Italy. Only in few locations the effect of the number of classes results to be significantly different from zero. In the area of Rome, in the Venice province and in Umbria the number of classes has a positive impact on school performance, whereas in the north areas of Campania, Apulia and

Calabria and in the east provinces of Sicily the number of classes is negatively associated with school performance. Only in Sardinia the number of buildings has a consistent significant negative effect on the performance. The percentage of females results to be positive associated to school performance, only in the area of Rome the mean performances of a few schools are negatively affected by the percentage of females. In the entire Calabria and in several area of North Italy the presence of foreign students affects the mean school performance negatively; in particular, in Calabria the magnitude of the coefficients is higher than in the rest of Italy. The schools with high values of the repeating students' coefficients are located in the province of Lecce, Siracusa, in the metropolitan area of Rome and in Tuscany.

To summarize the results of the GWR and to identify clusters of schools with performances homogeneously affected by the analysed factors, the k-means clustering has been applied on the regression coefficients matrix. The comparison between 30 different indices leads to the identification of seven as the optimal number of clusters. Table 2 shows the summary statistics for each clusters, in particular we have reported the mean and the percentage of significant coefficients in correspondence of each variable. It can be noticed that the WLE in G05 and ESCS have a predominant role in the interpretation of the clusters. The use of the k-means has facilitated the reading of the results since it is now possible to identify easily which other factors play a key role in correspondence of a specific geographic area (cluster).

Table 3. Summary statistics for clusters

Clusters	Statistics	WLE	ESCS	Number buildings	Number classes	Female (%)	Foreign (%)	Repeating (%)
1	**Mean**	0,311	0,362	0,037	0,038	0,022	-0,062	-0,168
	% Pv < 0.05	100,00%	100,00%	2,01%	30,92%	10,54%	23,19%	90,76%
2	**Mean**	0,245	0,439	0,038	0,030	0,046	-0,128	-0,083
	% Pv < 0.05	99,40%	100,00%	7,35%	5,84%	34,14%	63,54%	31,02%
3	**Mean**	0,309	0,320	-0,038	0,062	0,090	-0,117	-0,048
	% Pv < 0.05	99,85%	100,00%	0,77%	17,62%	69,09%	57,81%	0,77%
4	**Mean**	0,166	0,550	-0,001	-0,058	0,074	-0,323	-0,017
	% Pv < 0.05	100,00%	100,00%	17,45%	32,38%	54,19%	94,97%	1,68%
5	**Mean**	0,142	0,559	0,022	-0,016	0,064	-0,088	-0,062
	% Pv < 0.05	98,96%	100,00%	3,55%	5,62%	23,37%	1,48%	31,51%
6	**Mean**	0,298	0,249	-0,011	0,013	0,058	-0,176	-0,081
	% Pv < 0.05	99,73%	100,00%	14,97%	0,00%	31,28%	81,55%	31,02%
7	**Mean**	0,179	0,497	0,078	-0,069	0,069	0,038	-0,149
	% Pv < 0.05	99,59%	100,00%	72,65%	52,86%	40,82%	3,27%	73,88%

Map in Figure 3 reports the three clusters that represent almost completely the schools located in the South of Italy, covering the 25,85% of Italian schools. The radar plot highlights that focusing on the WLE in G05 and the ESCS these clusters result to be very similar. In all three clusters, the ESCS effect ranges only from 0,497 to 0,550 and its value is greater than the national average (0,40). On the other hand, the effect of the "input score" (which varies between 0,142 and 0,179) is less than the national one (0,248). This means that in the South is more important the family background instead of the previous competencies of the students. In the red cluster (Cluster 7) the 73,88% of the schools is significantly influenced by the presence of repeating students, that has on average a greater negative effect on the performance (-0,149) than the national average (-0,09). The presence of foreign students has a significant and negative effect on the performance for the 94,97% of the schools located in Cluster 4, which covers Calabria and Basilicata. For these schools the coefficient results to be on average (-0.323) three times higher than the national average (-0,126).

The behaviour of the schools belonging to Cluster 1 and Cluster 3 (Figure 4) is very similar; both have a positive effect of the previous competencies greater than the national average (0,31 towards 0,25), whereas the socio-economic background effect is less than the national average. In contrast to the reality described for the South (Figure 3), in these territories it is very important for the school what the student knows at the beginning of the lower secondary school and not his socio-economic status. Another important thing, for the blue cluster that covers part of the Tyrrhenian coast, is the negative effect, greater than the national average, of the presence of foreign students in the school (-0,168 towards -0,09).

Finally, the last two clusters include a particular section of the national territory, Alps, Friuli, Veneto, the Po valley, the Apennines and Sardinia. These two clusters are quite similar considering the effect of the previous competencies and the negative impact of the presence of foreign students, which results to be around the national average for both variables. On the other hand, an opposite behaviour of the family background is observed: for schools in Cluster 2 (in orange) the ESCS effect is greater than the national average, whereas in Cluster 6 (in yellow) the ESCS effect results to be less than the national average.

Figure 3. Spatial position and radar plot of school clusters in South Italy.

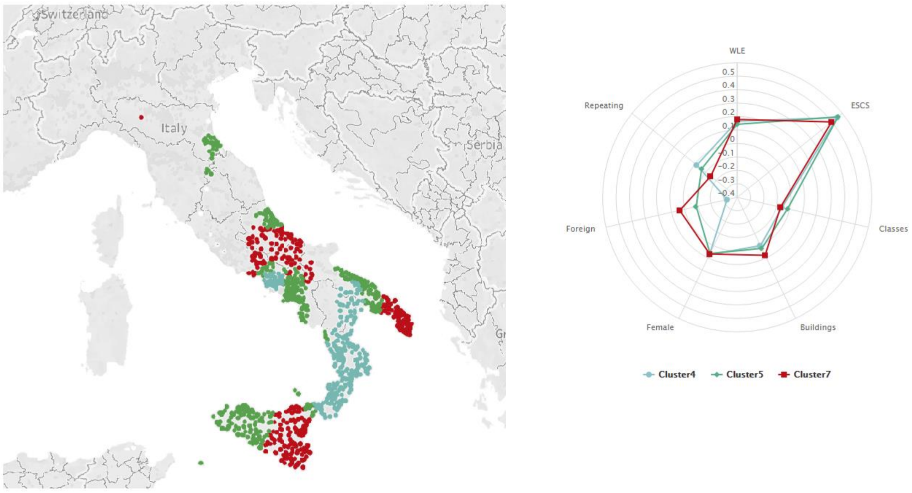


Figure 4. Spatial position and radar plot of school clusters in coastal regions.

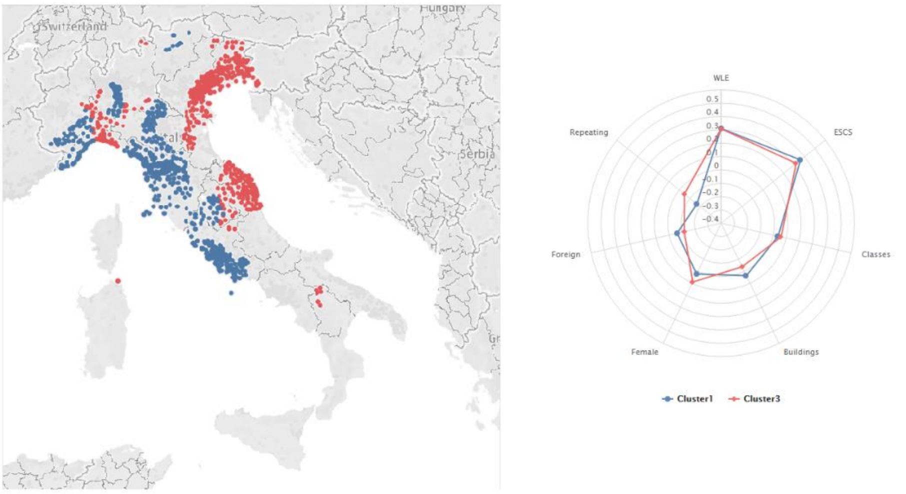
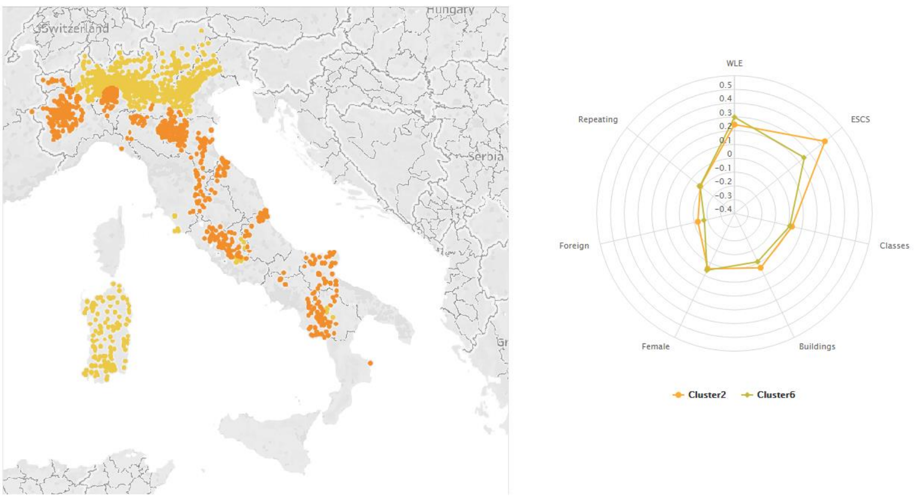


Figure 5. Spatial position and radar plot of school clusters in Italy.



Conclusions

The study of the territorial differences in educational achievement is a widely debated topic, in particular in Italy for the presence of the well-known strong regional disparities. In this study, our aim is to investigate the extent of the spatial disparities in the relationship between the academic achievement and some school-level factors moving beyond the regional administrative confines, in order to identify new spatial patterns.

We exploited standardized tests in Italian Language administered by INVALSI in 2017-2018 focusing on 8th grade students. As school-level factors we use a set of socioeconomic and demographic variables. The novelty of our approach in the study of the geographically disparities is the combined use of GWR, that allow to examine the spatially varying relationship, and of k-means clustering method to classify schools into homogeneous regions based on local regression patterns.

The spatial analysis of the relationship between academic achievement and school-level factors outlined a fragmented reality. The predictors of academic achievement are spatially non-stationary and using the k-means clustering we identified 7 school clusters that are homogeneous with respect to the factors' effect on school performance. Each cluster has been characterized geographically and in relation to the intensity of predictors statistically significant in the area. It is well-known that the competencies of the students at the beginning of the secondary school and the family socio-economic status play a key role, but we have found of particular interest and importance, in order to understand the complex Italian reality, how the effects of these variables changes with respect to the geographical position of the school. Focusing on South Italy, the schools' mean performances are strongly affected by the socio-economic status. On the other hand, the schools that belong to part of the Tyrrhenian and Adriatic coast are able to moderate the socio-economic differences of students and school's mean performances results to be affected mostly by students' competences at the beginning of the secondary school.

This work allows to visualize the Italian situation going beyond the classic territorial definition and offers an in-depth analysis of the Italian education system. The identification of new spatial clusters is a useful tool to differentiate supports for schools on the basis of their unique specific needs and is a first step to understand how to address a more contextualized policy response to educational needs in Italy.

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